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Innovation and copyright infringement: The Case of Commercial Piracy and End-user Piracy

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Abstract:

The purpose of this paper is to analyse the question, whether copyright infringement of digital products like software commonly labelled as piracy impedes innovation. We find the answer depends on the nature of piracy i.e. whether it is end-users or commercial piracy. For end user piracy, copyright infringement does not necessarily impede innovation; in fact it can be shown that it encourages innovation when the pirates are active. However, for commercial piracy, it always impedes innovation which has negative implications on the overall welfare of the society. We show under what conditions the government intervention through IPR protection strategy (like monitoring and imposing a fine to the pirate) can support the copyright holder for higher level of innovation. We find the socially optimally monitoring rate for the government that result in maximum innovation for the copyright holder.

JEL Classification: D21, D43, L13, L21, L26, O3.

Keywords: Innovation, piracy, monitoring, social welfare

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1. Introduction

The purpose of this paper is to analyse the question, whether copyright infringement of digital products like software commonly labeled as piracy impedes innovation. The answer to this question lies on the definition of innovation and on the type of piracy which can be either in the form of end-user piracy where copying is done for personal consumption or in the form of commercial piracy where copyiers illegally sell the copied product thereby competing with the copyright holder.

One way to perceive innovation of a new product is to consider a research and development (R&D) investment that leads to the development of product with a fixed quality. In this case both end-user and comercial piracies result in a reduced incentive to innovate measured by the difference between the revenue and the R&D investment. This is because copying results in a fall in the copyright holder's revenue, which reduces the net profit from R&D. Banerjee and Chatterjee (2010) has shown this in the context of a single innovating firm that faces technological uncertainty in the sense that the success in innovating the product is a random function of the R&D investment.

The alternative way to view innovation to consider a "continuous" process where the degree of innovation is measured by the quality of the product developed. In this context, Lahiri and Dey (2012) has shown in the context of end-user piracy that an increase in piracy measured by the quality of the copied product increases the quality of innovation, that is the incentive to innovate. This is because end-users face a fixed cost of copying. Consequently, an increase in the copied product leads to an increase in the quality of the copyright holder's product because it creates a higher product differentiation, that can recover some of the loss in their profit from piracy.

Using the continuous concept of innovation we consider the case of commercial piracy. In contrast to the case of end-user piracy we show that an increase in the quality of the copied product reduces the copyright holder's product quality and tus the incentive to innovate. This is because, in the case of commercial piracy, the copier, hereafter labeled as the pirate, competes with the copyright holder and adjusts his price in response to the changes in price and quality of the copyright holder's product. Thus in response to an increase in the pirate's product quality, it is optimal for the copyright holder to reduce its product quality thereby bringing it closer to that of the pirate. This is because an increase in the pirate's product quality results in an increase in its market share. In order to at least maintain its market share, the copyright holder reduces its product quality and price, therby recovering some of its lost profit. This outweighs the gain from higher product quality (and higher price) and lower market share.

We also perform the social welfare analysis and show that if the legally instituted fine is below a certain level then it is not socially optimal for the government to monitor piracy. Only when it this fine is above a certain critical level then monitoring is socially optimal and piracy is deterred. Increases in fine beyond this level increases the social welfare. However, it is never socially optimal to allow the quality level which is the same as that in the absence of piracy, the pure monopoly level outcome.

2. End-user piracy

There is a continuum of consumers indexed by θ, which represents the consumers' valuation of the product. We assume that θ follows a uniform distribution and lies in the interval $\theta \in [0,1]$. Each consumer is assumed to use at most only one unit of a product, which can either be the original or the copied one. A consumer enjoys of θQ from the consumption of the monopolist's product and $q\theta Q$ from the consumption of the copied products. Thus the utility of a type-θ consumer as in Banerjee (2003) is as follows.

$$U(\theta)=\begin{cases}\theta Q-p_m,\text{if the consumer uses the monopolist's product,}\\ q\theta Q-r,\ \text{if the consumer uses the copied product,}\\ 0,\qquad\qquad\qquad\qquad\qquad\qquad \text{otherwise.}\end{cases} \tag{1}$$

We consider the case where a copier makes copies of the monopolist's product for personal consumption. The game played between the monopolist and the copiers consist of the following stages.

Stage 1: Monopolist chooses the product quality Q and the price p_m.

Stage 2: The consumers make their consumption decision. If they copy they incur a copying cost r.

2.1 : Suppose $p_m > \dfrac{r}{q}$ holds.

In this case there is copying taking place and the demand for both products are as given in equation (2). The monopolist's profit is as follows.

$$\pi_m = p_m D_m - c(Q) = \left(p_m - \frac{p_m^2 - rp_m}{Q(1-q)}\right) - \frac{Q^2}{2} \tag{5}$$

Maximizing profit with respect to p_m and Q yields the following first order conditions.

$$\begin{aligned}&\frac{d\pi_m}{dp_m} = 1 - \frac{2p_m - r}{Q(1-q)} = 0 \Rightarrow p_m = \frac{Q(1-q)+r}{2},\\ &\frac{d\pi_m}{dQ} = \frac{p_m{}^2 - rp_m}{Q^2(1-q)} - Q = 0.\end{aligned} \tag{6}$$

Substituting $\dfrac{d\pi_m}{dp_m}=0$ in $\dfrac{d\pi_m}{dQ}=0$ we get

$$p_m\,(p_m - r) = Q^3(1-q) \Rightarrow \frac{Q^2(1-q)^2 - r^2}{4} = Q^3(1-q)$$

The sufficient condition for maximization is given in (7).

$$\frac{d^2\pi_m}{dp_m{}^2}<0,\frac{d^2\pi_m}{dQ^2}<0,|D|=\begin{vmatrix}\frac{d^2\pi_m}{dp_m{}^2} & \frac{d}{dQ}\left(\frac{d\pi_m}{dp_m}\right)\\ \frac{d}{dp_m}\left(\frac{d\pi_m}{dQ}\right) & \frac{d^2\pi_m}{dQ^2}\end{vmatrix}>0. \tag{7}$$

Now $\frac{d^2\pi_m}{dp_m{}^2}=-\frac{2}{Q(1-q)}<0,\frac{d^2\pi_m}{dQ^2}=\frac{-2(p_m{}^2-rp_m)}{Q^3(1-q)}-1<0,$ and

$\frac{d}{dQ}\left(\frac{d\pi_m}{dp_m}\right)=\frac{d}{dp_m}\left(\frac{d\pi_m}{dQ}\right)=\frac{2p_m-r}{Q^2(1-q)}>0.$ So,

$|D|=\frac{-r^2}{Q^4(1-q)^2}+\frac{2}{Q(1-q)}=\frac{2Q^3(1-q)-r^2}{Q^4(1-q)^2}>0$. Now

Now let us perform the comparative static analysis with respect to q and r. total differentiation of the system of equations in (6) with respect to q, Q and p_m gives us the following.

$$\begin{bmatrix}\frac{d^2\pi_m}{dp_m{}^2} & \frac{d}{dQ}\left(\frac{d\pi_m}{dp_m}\right)\\ \frac{d}{dp_m}\left(\frac{d\pi_m}{dQ}\right) & \frac{d^2\pi_m}{dQ^2}\end{bmatrix}\begin{bmatrix}\frac{dp_m}{dq}\\ \frac{dQ}{dq}\end{bmatrix}=\begin{bmatrix}-\frac{2p_m-r}{Q(1-q)^2}\\ \frac{p_m^2-rp_m}{Q^2(1-q)^2}\end{bmatrix}0 \tag{8}$$

Applying Cramer's rule we solve for $\frac{dp_m}{dq}$ and $\frac{dQ}{dq}$ which are as follows.

$\frac{dp_m}{dq}=\frac{\begin{vmatrix}-\frac{2p_m-r}{Q(1-q)^2} & \frac{d^2\pi_m}{dQdp_m}\\ -\frac{p_m^2-rp_m}{Q^2(1-q)^2} & \frac{d^2\pi_m}{dQ^2}\end{vmatrix}}{|D|}$. From the second order condition we know the sign of the denominator to be positive. So the sign of $\frac{dp_m}{dq}$ depends on the sign of the numerator. Since

$\frac{d}{dQ}\left(\frac{d\pi_m}{dp_m}\right)=\frac{d}{dp_m}\left(\frac{d\pi_m}{dQ}\right)=\frac{2p_m-r}{Q^2(1-q)}>0$ and $\frac{d^2\pi_m}{dQ^2}=\frac{-2(p_m{}^2-rp_m)}{Q^3(1-q)}-1<0,$ hence the sign of the numerator is positive. Thus $\frac{dp_m}{dq}>0$. Similarly,

$\frac{dQ}{dq}=\frac{\begin{vmatrix}\frac{d^2\pi_m}{dp_m{}^2} & -\frac{2p_m-r}{Q(1-q)^2}\\ \frac{d^2\pi_m}{dp_m dQ} & -\frac{p_m^2-rp_m}{Q^2(1-q)^2}\end{vmatrix}}{|D|}>0$ because $\frac{d}{dQ}\left(\frac{d\pi_m}{dp_m}\right)=\frac{d}{dp_m}\left(\frac{d\pi_m}{dQ}\right)=\frac{2p_m-r}{Q^2(1-q)}>0$ and $\frac{d^2\pi_m}{dp_m{}^2}<0$.

Similarly, the comparative static analysis with respect to *r* yields the following results.

$$\begin{bmatrix}\frac{d^2\pi_m}{dp_m{}^2} & \frac{d}{dQ}\left(\frac{d\pi_m}{dp_m}\right)\\ \frac{d}{dp_m}\left(\frac{d\pi_m}{dQ}\right) & \frac{d^2\pi_m}{dQ^2}\end{bmatrix}\begin{bmatrix}\frac{dp_m}{dr}\\ \frac{dQ}{dr}\end{bmatrix}=\begin{bmatrix}\frac{-1}{Q(1-q)}\\ \frac{p_m}{Q^2(1-q)}\end{bmatrix} \tag{9}$$

Now $\frac{dp_m}{dr}=\frac{\begin{vmatrix}-\frac{1}{Q(1-q)^2} & \frac{d^2\pi_m}{dQdp_m}\\ \frac{p_m}{Q^2(1-q)} & \frac{d^2\pi_m}{dQ^2}\end{vmatrix}}{|D|}$ the sign of which again depends on the sign of the numerator. Up on simplification $\begin{vmatrix}-\frac{1}{Q(1-q)^2} & \frac{d^2\pi_m}{dQdp_m}\\ \frac{p_m}{Q^2(1-q)} & \frac{d^2\pi_m}{dQ^2}\end{vmatrix}=\frac{-p_m r}{Q^4(1-q)^2}<0$. Thus $\frac{dp_m}{dr}<0$.

Following the same methodology we get $\frac{dp_m}{dQ}<0$.

Case 2 (No Piracy - Pure Monopoly)

In the case of pure monopoly we only need to consider the constraint IR-M for which the marginal consumer is θ_1 who is indifferent between purchasing the original product and buying nothing. So the demand and the profit functions are $D_m\big|_{monopoly} = 1-\theta_1 = 1-\dfrac{p_m}{Q}$ and $\pi_m\big|_{monopoly} = \left(p_m - \dfrac{p_m^2}{Q}\right) - \dfrac{Q^2}{2}$. Maximization of this with respect to p_m and Q yields

$$p_m\Big|_{monopoly} = \frac{1}{8},\ Q\Big|_{monopoly} = \frac{1}{4} \text{ and } \pi_m\Big|_{monopoly} = \frac{1}{32}. \tag{10}$$

3. Analysis of commercial piracy

Let us consider the market for a digital/information good, like software. There is a single innovating firm (a monopolist), who invests in R&D to develop a product of quality Q at a cost $c(Q) = \dfrac{Q^2}{2}$. There is a firm (pirate) who copies the original product and illegally sells it in the market thereby competing with the copyright holder. As explained in detail in Takeyama (1994), Banerjee (2003), and Lahiri and De (2012), the pirated product is an inferior substitute of the monopolist's product so the quality of the pirated product is given by qQ where $q \in (0,1)$.[3] The government is responsible for monitoring the illegal activities of the pirate who if detected which occurs with probability α is penalized with a fine *F*.

The government's monitoring cost is $c(\alpha) = \dfrac{\alpha^2}{2}$. The extensive form game played between the players is as follows.

Stage 1: The government chooses a monitoring rate α.

[3] q can be interpreted as an exogenous index of the poor quality of the pirated product. We set this bound to ensure that the profits are not indeterminate.

Stage 2: The monopolist chooses either an *entry-allowing* (*ea*) or an *entry-deterring* (*ed*) quality-price pair, represented by (Q, p_m).

Stage 3: The pirate makes its entry decision. If it chooses to enter then it chooses a price p_c.

Stage 4: The buyers decide either to buy the original or the pirated product, or nothing.

There is a continuum of consumers indexed by θ, which represents the consumers' valuation of the product. We assume that θ follows a uniform distribution and lies in the interval $\theta \in [0,1]$. Each consumer is assumed to use at most only one unit of a product, which can either be the original or the copied one. A consumer enjoys of θQ from the consumption of the monopolist's product and $q\theta Q$ from the consumption of the copied products. Thus the utility of a type-θ consumer as in Banerjee (2003) is as follows.

$$U(\theta) = \begin{cases} \theta Q - p_m, \text{if the consumer uses the monopolist's product,} \\ q\theta Q - p_c, \text{ if the consumer uses the copied product,} \\ 0, \qquad\qquad\qquad\qquad\qquad\qquad \text{otherwise.} \end{cases} \tag{1}$$

Consumer's Decision Problem. Consumers decide whether to consume the monopolist's product or the copied product depending on he *individual rationality* (IR) and *incentive compatibility* (IC) constraints. A consumer consumes the monopolist's product if the following IR and IC conditions are satisfied.

$$\theta Q - p_m \geq 0 \Rightarrow \theta \geq \frac{p_m}{Q} = \theta_1 \tag{IR-M}$$

$$\theta Q - p_m \geq q\theta Q - p_c \Rightarrow \theta \geq \frac{p_m - p_c}{Q(1-q)} = \theta_2 \tag{IC-M}$$

Similarly, a consumer buys the pirated product if the following IR and IC conditions hold.

$$q\theta Q - p_c \geq 0 \Rightarrow \theta \geq \frac{p_c}{qQ} = \theta_3 \tag{IR-C}$$

$$\theta Q - p_m < q\theta Q - p_c \Rightarrow \theta < \frac{p_m - p_c}{Q(1-q)} = \theta_2 \tag{IC-C}$$

Now there are two possibilities.

(1) $\theta_1 > \theta_3 \Rightarrow p_m \geq \frac{p_c}{q}$ This implies that that the price of the monopolist's product software exceeds the effective price of the pirated product, which is the price per unit quality of the pirated product. If $\theta_1 > \theta_3 \Rightarrow p_m \geq \frac{p_c}{q}$ holds then $\theta_2 > \theta_1 > \theta_3 > 0$. We see that the market is shared between the monopolist's and the copied products. In this case the demand functions are as given in equation (2).

$$\begin{aligned} D_m &= 1-\theta_2 = 1-\frac{p_m - p_c}{Q(1-q)} \\ D_c &= \theta_2 - \theta_3 = \frac{qp_m - p_c}{qQ(1-q)} \end{aligned} \tag{2}$$

(2) If $\theta_1 \leq \theta_3$ holds, which implies that $p_m \leq \frac{p_c}{q}$, then no user finds it optimal to buy the pirated product and there are only legal users who buys it from the monopolist, that is, $D_c = 0$. In this case the demand for the monopolist's product is given in equation (3).

$$D_m = 1-\theta_1\Big|_{D_c=0} = 1-\frac{p_m}{Q} \tag{3}$$

So the demand for the monopolist's product can be concisely written as

$$D_m = \begin{cases} 1-\theta_2 = 1-\dfrac{p_m - p_c}{Q(1-q)}, if\, \theta_1 \geq \theta_3, \\ 1-\theta_1 = 1-\dfrac{p_m}{Q},\ if\, \theta_1 \leq \theta_3. \end{cases} \tag{4}$$

We first show that in the case of commercial piracy it is only possible to have $\theta_1 > \theta_3 \Rightarrow p_m \geq \frac{p_c}{q}$ in equilibrium. This implies that the inequality $\theta_2 > \theta_1 > \theta_3 > 0$ also holds, and the market is shared between the monopolist and the pirate.

Suppose $\theta_1 \geq \theta_3$ holds and the monopolist chooses the *entry-allowing* Q and p_m. If the pirate chooses to enter, then from the demand functions given in equation (2) we get the profit functions as follows.

$$\begin{aligned} \pi_m &= p_m D_m - c(Q) = p_m - \frac{p_m^2 - p_m p_c}{Q(1-q)} - \frac{Q^2}{2} \\ \pi_c &= p_c D_c - \alpha F = \frac{q p_m p_c - p_{c\,c}^{2}}{qQ(1-q)} - \alpha F \end{aligned} \tag{5}$$

The pirate's reaction function is $p_c = \frac{qp_m}{2}$. Substituting this in the monopolist's profit function and maximizing it with respect to Q and a price p_m we get the price and the quality for the *ea-strategy*, which are,

$$(p_m^{ea}, Q^{ea}) = \left(\frac{(1-q)^2}{2(2-q)^2}, \frac{(1-q)}{2(2-q)} \right). \tag{6}$$

Observe from (12) that $2(Q^{ea})^2 = p_m^{ea}$. The pirate's equilibrium price is $p_c^{ea} = \frac{q(1-q)^2}{4(2-q)^2}$. The monopolist's and the pirate's profits in equilibrium and the consumer surplus (CS^{ea}) are,

$$\begin{aligned} \pi^{ea}{}_m &= \frac{(1-q)^2}{8(2-q)^2}, \\ \pi_c &= \frac{q(1-q)^2}{16(2-q)^3} - \alpha F, \\ CS^{ea} &= \frac{(3+q)Q^{ea}}{8} - \frac{(2+q)(Q^{ea})^2}{2} + \frac{q(Q^{ea})^3}{2} \end{aligned} \tag{7}$$

From the expressions of p_m^{ea} and p_c^{ea} we see that the condition $\theta_1 > \theta_3 \Rightarrow p_m > \frac{p_c}{q}$ always holds in equilibrium. This is because p_c always directly adjusts to changes in p_m following the reaction function. This is in contrast to the case of the end-user piracy where the cost of making copies is fixed and the monopolist can choose to set a price such that $\theta_1 \leq \theta_3$ which eliminates copying.[4]

Thus in the case of commercial piracy, entry-deterrence is executed differently from that in the case of end-user piracy, which is as follows. We substitute the pirate's reaction function in his profit function. The monopolist then chooses a price p_m (or a quality *Q*) such that for any given *Q* (or p_m), the pirate's profit is zero, which prevents his entry. This process gives us $Q = \frac{qp_m^2}{4\alpha F(1-q)}$. Substituting this *Q* into π_m and maximizing it with respect to p_m yields the following price and quality for the *ed-strategy*;

$$(p_m^{ed}, Q^{ed}) = \left(\frac{2(1-q)^{\frac{2}{3}}\alpha^{\frac{2}{3}}F^{\frac{2}{3}}}{q^{\frac{2}{3}}}, \frac{(1-q)^{\frac{1}{3}}\alpha^{\frac{1}{3}}F^{\frac{1}{3}}}{q^{\frac{1}{3}}} \right). \tag{8}$$

Notice that $2(Q^{ed})^2 = p_m^{ed}$. Using this we can write the monopolist's profit and the consumer surplus (CS^{ed}) for the *ed-strategy* in equilibrium as

$$\begin{aligned} \pi_m^{ed} &= \frac{3(Q^{ed})^2}{2} - 4(Q^{ed})^3, \\ CS^{ed} &= \frac{Q^{ed}}{2} - 2(Q^{ed})^2 + 2(Q^{ed})^3. \end{aligned} \tag{9}$$

Differentiating π_m^{ed} with respect to α gives us; $\frac{d\pi_m^{ed}}{d\alpha} = 3Q^{ed}(1-4Q^{ed})\frac{dQ^{ed}}{d\alpha}$. Now $\frac{dQ^{ed}}{d\alpha} > 0$. So $\frac{d\pi_m^{ed}}{d\alpha} \geq 0$ for $Q^{ed} \leq \frac{1}{4} = Q^{monopoly}$ which is the equilibrium quality in the monopoly case. Let $\alpha_{\max}$ be the monitoring rate at which the monopoly outcome is restored. Equating $Q^{ed} = Q^{monopoly} = \frac{1}{4}$ we get $\alpha_{\max} = \frac{q}{64F(1-q)}$. Monitoring beyond $\alpha_{\max}$ will not

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[4] In Lahiri and De the fixed cost of making copies is *r*, which does not adjust to changes in p_m. Thus the monopolist can choose a price such that consumers do not copy.

alter the monopoly outcome hence, for the entire analysis the relevant monitoring rate range is $\alpha \in [0, \alpha_{\max}]$ where π_m^{ed} is increasing in α.

Lemma 1 provides a comparison the of π_m^{ea} and π_m^{ed} with respect α. The proof is given in the Appendix.

Lemma 1. *There exists a unique* α *, say* $\hat{\alpha}$ *, at which; (i)* $p_m^{ea} = \pi_m^{ed}(\hat{\alpha})$ *and (ii)* $Q^{ed}(\hat{\alpha}) < Q^{ea}$.

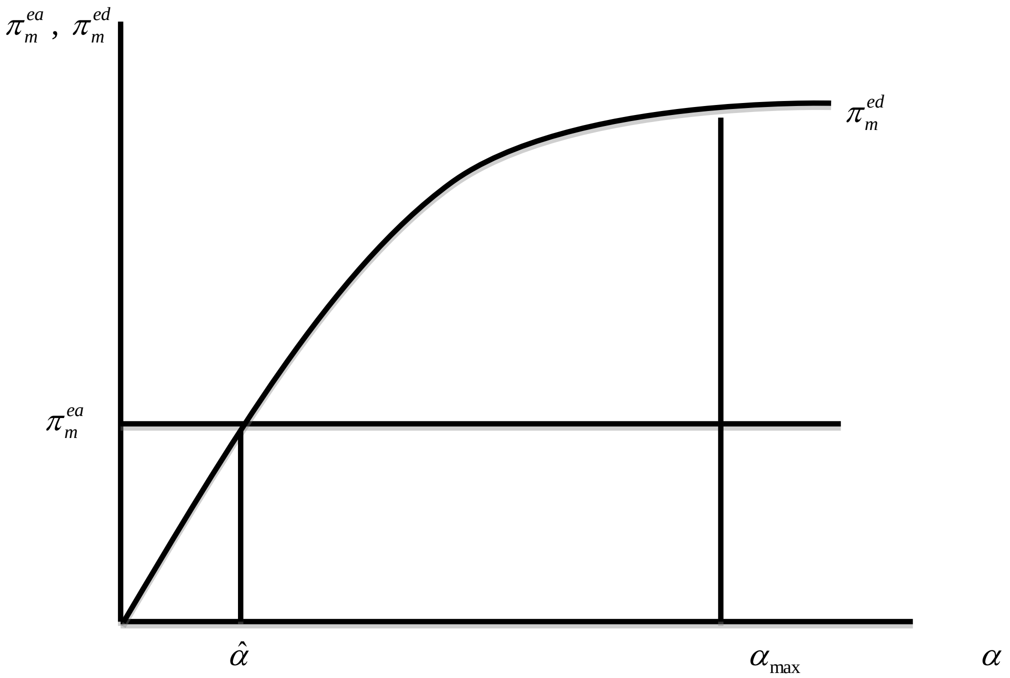

Figure 1. Comparison of the comparative statics of π_m^{ea} and π_m^{ed} with respect to α

The implication of Lemma 1 as evident from Figure 1 is that the *ea-strategy* dominates the *ed-strategy* in the range $\alpha \in [0, \hat{\alpha})$ and the latter is weakly dominant in the range $\alpha \in [\hat{\alpha}, \alpha_{\max}]$. We use Lemma 1 and Figure 1 to characterise the equilibrium price p_m^* and quality Q^* in the case of commercial piracy. This result is summarized in Proposition 1.

Proposition 1. *The equilibrium price-quality pair for different levels of monitoring is*

$$(p_m^*, Q^*) = \begin{cases} (p_m^{ea}, Q^{ea}) \ for \alpha \in [0, \hat{\alpha}), \\ (p_m^{ed}, Q^{ed}) \ for \alpha \in [\hat{\alpha}, \alpha_{\max}]. \end{cases}$$

The proof of Proposition 1 follows from Lemma 1 that *ea-strategy* is dominant in the range $\alpha \in [0, \hat{\alpha})$ *ed-strategy* is weakly dominant in the range $\alpha \in [\hat{\alpha}, \alpha_{\max}]$. Notice that $\alpha_{\max} - \hat{\alpha} > 0$. We now proceed to perform the comparative static analysis with respect to *q* and *F*, which will help us to do the social welfare analysis and determine the socially optimal monitoring rate.

3.1. Comparative static analysis with respect to *q*: Effect of piracy on innovation.

To understand the effect of a change in *q* on we first determine the expressions for θ_1, θ_2 and θ_3 for the *ea* and *ed* strategies. Substituting $(p_m^{ea}, Q^{ea}) = \left(\frac{(1-q)^2}{2(2-q)^2}, \frac{(1-q)}{2(2-q)} \right)$ and $p_c^{ea} = \frac{q(1-q)^2}{4(2-q)^2}$ we get $\theta_1^{ea} = \frac{1-q}{2-q}, \theta_2^{ea} = \frac{1}{2}$ and $\theta_3^{ea} = \frac{1-q}{2(2-q)}$. Clearly θ_1^{ea} and θ_3^{ea} are decreasing in *q*, while θ_2^{ea} is a constant. This means that the demand for the monopolist's product, which is $D_m^{ea} = 1 - \theta_2^{ea} = \frac{1}{2}$ is invariant to changes in *q*, while $D_c^{ea} = \theta_2^{ea} - \theta_3^{ea} = \frac{1}{2(2-q)}$ is increasing in *q*. These findings are used for Lemma 2 which is as follows. For the *ed-strategy*, $D_m^{ed} = 1 - \theta_1^{ed} = 1 - \frac{p_m^{ed}}{Q^{ed}} = 1 - 2Q^{ed}$. Now

$$\frac{dD_m^{ed}}{dq} = 1 - \theta_1^{ed} = 1 - \frac{p_m^{ed}}{Q^{ed}} = 1 - 2Q^{ed}$$

Lemma 2. *In case of the ea-strategy, an increase in q increases the pirate's market share, which is given by* $s = \frac{D_c^{ea}}{D_m^{ea} + D_c^{ea}}$. *For the ed-strategy, an increase in q increases the demand for the monopolist's product, which is* D_m^{ed}.

Thus an increase in *q* increases the pirate's market share, which implies that piracy increases. Proposition to 2 summarizes the effect of a change in *q* on Q^* which is a measure of the effect of a change in piracy on the incentive to innovate.

Proposition 2. *An increase in piracy measured by an increase in q reduces the incentive to innovate that is, reduces* Q^*.

The proof of Proposition 2 follows from the fact that $\frac{dQ^{ea}}{dq} = \frac{-1}{(2-q)^2} < 0$ and $\frac{dQ^{ed}}{dq} = \frac{-1}{3q^2}\left(\frac{1-q}{q}\right)^{-\frac{2}{3}} < 0$. This result is in contrast to the effect of an increase in *q* on the quality of innovation in the case of end-user piracy. The intuition behind Proposition 2 and the difference between this result and that in the case of end-user piracy can be explained as follows.

In the case of commercial piracy the pirate can adjust its price in response to changes in the quality and price chosen by the monopolist. So when there is an increase in *q* the pirate's market share increases as shown previously. To prevent a slide in its market share, the monopolist reduces the quality of its product thus bringing its product closer to that of the pirate. Consequently the equilibrium price decreases. In the case of the *ed-strategy* since an increase in *q* lowers θ_3 who is the marginal consumer indifferent between buying the pirated product and buying nothing, a lower price and quality for the monopolist's product is required to deter the pirate's entry. This in turn increases the demand for the monopolist's product.

Thus the effect of an increase in q on innovation in the case of commercial piracy is negatively monotonic. In contrast Lahiri and De (2012) shows that the effect of a change in q on innovation is non-monotonic.

This thus provides a justification as to why different government agencies is in particular focussing on commercial piracy and appealing to different countries to take stricter measures against it. So we next turn to the question of socially optimal enforcement polices and its effect on innovation. This in our paper the enforcement policy is captured by the monitoring rate. Such an analysis requires us to do some comparative static analysis of $\hat{\alpha}$ and $\alpha_{\max}$ with respect to q and F. This result is stated in Lemma 3 and is diagrammatically represented in Figures 2 and 3.

Lemma 3. *(i) An increase in q increases* $\alpha_{\max}$ *. (ii) An increase in F reduces* $\hat{\alpha}$ *, and* $\alpha_{\max}$ *.*

The positive effect of an increase in q on $\alpha_{\max} = \dfrac{q}{64F(1-q)}$ is clear from the fact that $\dfrac{q}{(1-q)}$ is increasing in q. However, a change in q on has an ambiguous effect $\hat{\alpha}$. This ambiguity is because an increase in q reduces π_m^{ea} and π_m^{ed} .

The negative effect of a change in F on $\alpha_{\max}$ is evident from the expression $\alpha_{\max} = \dfrac{q}{64F(1-q)}$. The negative effect of F on $\hat{\alpha}$ is because of the upward shift of the π_m^{ed} - curve due to an increase in F ($\dfrac{d\pi_m^{ed}}{dF} = 3Q^{ed}(1-4Q^{ed})\dfrac{dQ^{ed}}{dF} \geq 0$ for $Q^{ed} \leq Q^{monopoly} = \dfrac{1}{4}$). Intuitively, an increase in F increases the pirate's entry cost (αF) thereby reducing the pirate's possibility of entry thus increasing π_m^{ed} . π_m^{ea} is invariant to changes in α and F. Consequently, an increase in F reduces $\hat{\alpha}$. This is diagrammatically represented in Figure 3.

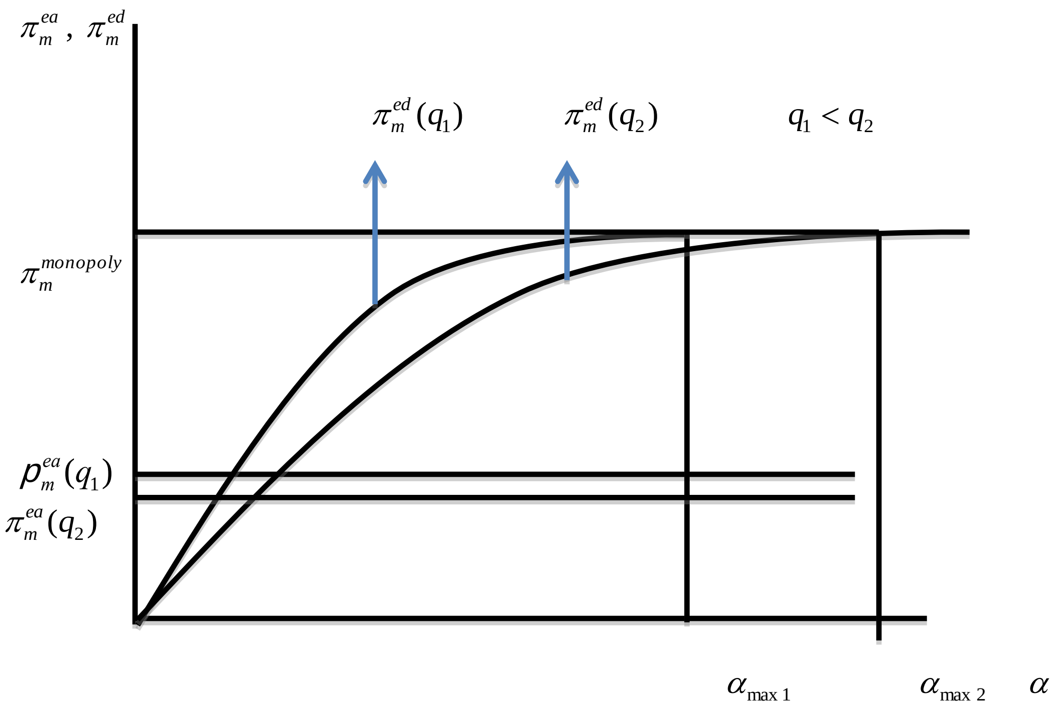

Figure 2. Effect of an increase in *q* on $\alpha_{\max}$

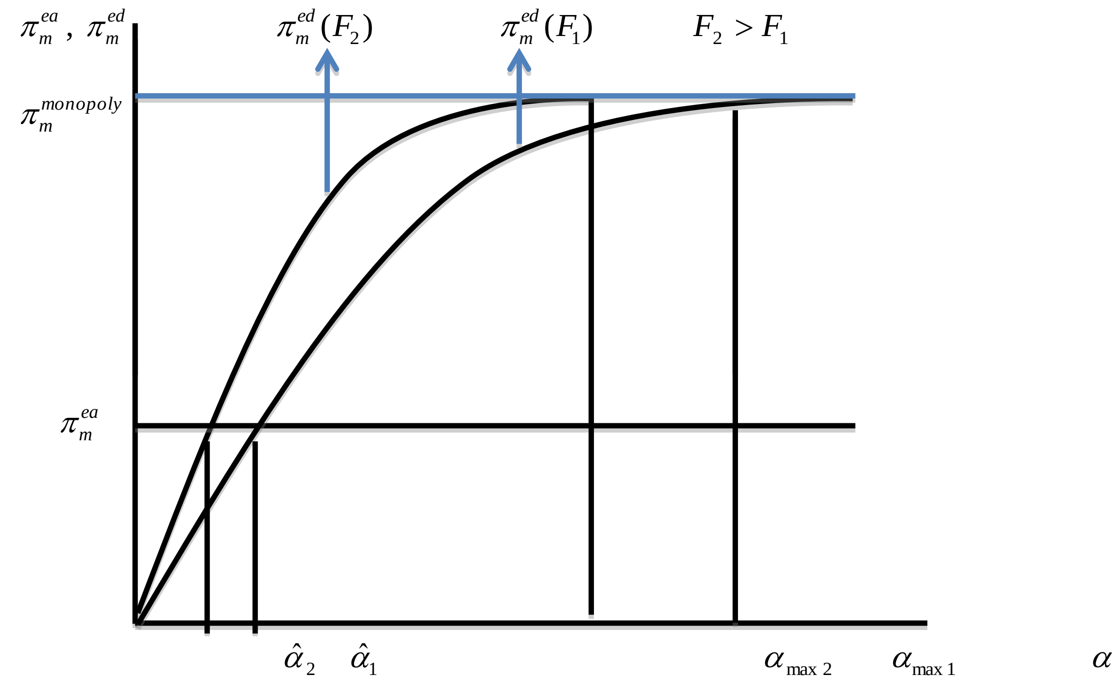

Figure 3. Effect of an increase in *F* on π_m^{ea} **,** π_m^{ed} **,** $\hat{\alpha}$ **and** $\alpha_{\max}$.

3.3. Social welfare analysis

From the above analysis we see that while an increase in *q* reduces Q^*, Q^*, π_m^{ed}, and increases $\alpha_{\max}$. An increase in *F* increases Q^{ed}, π_m^{ed} and reduces $\hat{\alpha}$ and $\alpha_{\max}$ leaving Q^* and Q^{ea} unchanged. We use these comparative static analysis results to examine the socially optimal monitoring rate. Social welfare is defined as the sum of the surplus of all agents in the model and is given in the following equation. We will use *CS* to denote the consumer surplus. Thus the social welfare function corresponding to the monopolist's strategy *i*, where $i \in \{ea, ed\}$ is as follows.

$$SW^i = \pi_m{}^i + \pi_c{}^i + CS^i + \alpha F - \frac{\alpha^2}{2} \tag{16}$$

From Lemma 1 and Figure 1 we know that the *ea-strategy* dominates the *ed-strategy* in the range $\alpha \in [0, \hat{\alpha})$ and the latter is weakly dominant in the range $\alpha \in [\hat{\alpha}, \alpha_{\max}]$. So the social welfare function (*SW*) for the relevant range of the monitoring for the relevant range of the monitoring rate, which is $\alpha \in [0, \alpha_{\max}]$, is,

$$SW = \begin{cases} SW^{ea}, \; for\, \alpha \in [0, \hat{\alpha}), \\ SW^{ed}, for\, \alpha \in [\hat{\alpha}, \alpha_{\max}]. \end{cases} \tag{17}$$

We first discuss the various properties of SW^{ea} and SW^{ed} which we will use to characterize the social welfare maximizing monitoring rate.

3.3.1. Analysis of SW^{ea}

Let α^{ea} denote the monitoring rate that maximizes SW^{ea}. Recall from equation (11) that the pirate's profit for the monopolist's *ea-strategy* in equilibrium is

$\pi_c = \dfrac{q(1-q)^2}{16(2-q)^3} - \alpha F$. Observe that the pirate does not enter if $\pi_c \le 0$. This no-entry

condition can be written as $\alpha \ge \overline{\alpha} = \dfrac{q(1-q)^2}{16F(2-q)^3}$.

Suppose $\alpha < \overline{\alpha}$ which means that the pirate enters. In this case since detection takes after the pirate has sold his product, the consumer surplus is unaffected by changes in the monitoring rate in the range $\alpha < \overline{\alpha}$. From equation (11) we know that π_m^{ea} is invariant to changes in α. The pirates expected fine αF being a transfer from the pirate to the government disappears from the social welfare function. Thus it is only the monitoring cost that appears in SW^{ea} in the range $\alpha < \overline{\alpha}$.

Now suppose $\alpha \geq \overline{\alpha}$ and the pirate does not enter. In this case, the monopolist's profit remains the same as given in equation (11). This is because any deviation to a Q above Q^{ea} will result in entry and is not optimal. This consumer surplus is less than when the pirate enters because the market size has reduced. Further, the cost of monitoring is now higher because α is now in the higher range $\alpha \geq \overline{\alpha}$. This and the analysis in the previous paragraph lead us to Lemma 4.

Lemma 4. $\alpha^{ea} = 0$ *maximizes* SW^{ea}.

3.3.2. Analysis of SW^{ed}

Let α^{ed} denote the monitoring rate that maximizes SW^{ed}. The consumer surplus for this strategy is,

$$CS^{ed} = \int_{\theta_1^{ed}}^{1} (\theta Q^{ed} - p_m^{ed}) d\theta = \frac{Q^{ed}}{2} - 2(Q^{ed})^2 + 2(Q^{ed})^3, \tag{18}$$

where $\theta_1^{ed} = \dfrac{p_m^{ed}}{Q^{ed}}$ as given in the IR-M constraint. Since the pirate's entry is deterred, the social welfare function is,

$$SW^{ed} = CS^{ed} + \pi_m^{ed} - \frac{\alpha^2}{2} = \frac{Q^{ed}}{2} - \frac{(Q^{ed})^2}{2} - 2(Q^{ed})^3 - \frac{\alpha^2}{2}. \tag{19}$$

Let α^{ed} be the monitoring rate that satisfies the first and the second order conditions which are $\frac{dSW^{ed}}{d\alpha} = 0 \ and \ \frac{d^2SW^{ed}}{d\alpha^2} < 0$.

$$\frac{dSW^{ed}}{d\alpha} = 0 \Rightarrow \left(\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right)\frac{dQ^{ed}}{d\alpha} - \alpha = 0$$
$$\Rightarrow \left(\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right)\left(\frac{A^{\frac{1}{3}}\alpha^{-\frac{2}{3}}F^{\frac{1}{3}}}{3}\right) - \alpha = 0$$
$$\Rightarrow \left(\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right)\left(\frac{Q^{ed}}{3\alpha}\right) - \alpha = 0 \tag{20}$$
$$\Rightarrow Q^{ed}\left(\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right) - 3\alpha^2 = 0,$$

where $A = \frac{(1-q)}{q}$ and $\frac{A^{\frac{1}{3}}\alpha^{-\frac{2}{3}}F^{\frac{1}{3}}}{3} = \frac{A^{\frac{1}{3}}\alpha^{\frac{1}{3}}F^{\frac{1}{3}}}{3\alpha} = \frac{Q^{ed}}{3\alpha}$ because $Q^{ed} = A^{\frac{1}{3}}\alpha^{\frac{1}{3}}F^{\frac{1}{3}}$.

The second order condition for maximization is

$$\frac{d^2SW^{ed}}{d\alpha^2} = \frac{d}{d\alpha}\left[\left(\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right)\left(\frac{Q^{ed}}{3\alpha}\right) - \alpha\right]$$
$$= \frac{\left(\frac{1}{2} - 2Q^{ed} - 18(Q^{ed})^2\right)}{3\alpha}\frac{dQ^{ed}}{d\alpha} - \frac{Q^{ed}\left(\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right)}{3\alpha^2} - 1$$
$$= \frac{\left(\frac{1}{2} - 2Q^{ed} - 18(Q^{ed})^2\right)}{3\alpha}\frac{Q^{ed}}{3\alpha} - \frac{Q^{ed}\left(\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right)}{3\alpha^2} - 1 \tag{21}$$
$$= \frac{Q^{ed}(Q^{ed} - 1)}{9\alpha^2} - 1 < 0,$$

because the highest possible quality is $Q^{ed} = \frac{1}{4}$ which is the quality under the pure monopoly situation. Hence the second order condition for maximization is satisfied.

From equation (20) observe that $\left(\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right) > 0$ must hold for the first order condition to be satisfied. Solving for this yields $Q^{ed} < 0.217$ for the first order condition to hold. Recall that at $\alpha = \alpha_{\max}$, $Q^{ed} = \frac{1}{4}$ which implies that $\alpha^{ed} < \alpha_{\max}$ since Q^{ed} is monotonically in α in the interval $\alpha \in [0, \alpha_{\max}]$. Next we perform the comparative static analysis of SW^{ed}, α^{ed}, $SW^{ed}(\alpha^{ed})$, and $Q^{ed}(\alpha^{ed})$ with respect to *F*. This will allow us to determine how the SW^{ed} curve, α^{ed}, the peak of the SW^{ed} curve which is given by $SW^{ed}(\alpha^{ed})$ and the optimal quality level at α^{ed} given by $Q^{ed}(\alpha^{ed})$ shifts or changes as F changes. The result is summarized in Lemma 4.

Lemma 5. *An increase in F:*

(i) increases SW^{ed} *for* $Q^{ed} < 0.217$ *and decreases* SW^{ed} *for* $Q^{ed} > 0.217$;

(ii) increases $SW^{ed}(\alpha^{ed})$;

(iii) increases α^{ed} *for* $Q^{ed} < 0.12$ *and decreases* α^{ed} *for* $Q^{ed} < 0.12$;

(iv) *increases* $Q^{ed}(\alpha^{ed})$ *since at* α^{ed}, $Q^{ed} < 0.217$.

Proof of Lemma 5.

$$(i)\frac{dSW^{ed}}{dF} = \left(\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right)\frac{dQ^{ed}}{dF}$$

$$= \left(\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right)\left(\frac{A^{\frac{1}{3}}\alpha^{\frac{1}{3}}F^{-\frac{2}{3}}}{3}\right) = \left(\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right)\left(\frac{Q^{ed}}{3F}\right)$$

Now $\frac{dSW^{ed}}{dF} > 0$ for $Q^{ed} < 0.217$ and $\frac{dSW^{ed}}{dF} < 0$ otherwise.

(*ii*) This follows from the fact that at α^{ed}, $Q^{ed} < 0.217$. Hence, an increase in *F* increases $SW^{ed}(\alpha^{ed})$.

(iii) To show $\frac{d\alpha^{ed}}{dF} < 0$ we perform the total differentiation of

$\frac{dSW^{ed}}{d\alpha} = \left(\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right)\frac{dQ^{ed}}{d\alpha} - \alpha = 0$ the solution of which is α^{ed}, with respect to α^{ed}

and F. That is, $\frac{\delta}{\delta\alpha^{ed}}\left(\frac{dSW^{ed}}{d\alpha^{ed}}\right)d\alpha^{ed} + \frac{\delta}{\delta F}\left(\frac{dSW^{ed}}{d\alpha^{ed}}\right)dF = 0 \Rightarrow \frac{d\alpha^{ed}}{dF} = -\frac{\frac{\delta}{\delta F}\left(\frac{dSW^{ed}}{d\alpha^{ed}}\right)}{\frac{\delta}{\delta\alpha^{ed}}\left(\frac{dSW^{ed}}{d\alpha^{ed}}\right)}$. The denominator is the same as that given in equation (21). So we need to evaluate the numerator to determine the sign of $\frac{d\alpha^{ed}}{dF}$. Using $\frac{dSW^{ed}}{d\alpha} = \left(\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right)\left(\frac{Q^{ed}}{3\alpha}\right) - \alpha$ from equation (20) we get

$$\frac{\delta}{\delta F}\left(\frac{dSW^{ed}}{d\alpha}\right) = \left(\frac{1}{2} - 2Q^{ed} - 18(Q^{ed})^2\right)\left(\frac{1}{3\alpha}\right)\frac{dQ^{ed}}{dF} = \left(\frac{1}{2} - 2Q^{ed} - 18(Q^{ed})^2\right)\left(\frac{1}{3\alpha}\right)\frac{Q^{ed}}{3F}.$$

Solving for $\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2 = 0$ we get $Q^{ed} = 0.12$. Therefore,

$\frac{\delta}{\delta F}\left(\frac{dSW^{ed}}{d\alpha}\right) = \begin{cases} > 0, for Q^{ed} < 0.12, \\ < 0, otherwise. \end{cases}$ So $\frac{d\alpha^{ed}}{dF} = \begin{cases} > 0, for Q^{ed} < 0.12, \\ < 0, otherwise. \end{cases}$ Now

$$\frac{d\alpha^{ed}}{dF} = -\frac{\frac{\delta}{\delta F}\left(\frac{dSW^{ed}}{d\alpha^{ed}}\right)}{\frac{\delta}{\delta\alpha^{ed}}\left(\frac{dSW^{ed}}{d\alpha^{ed}}\right)} = \frac{\left(\frac{1}{2} - 2Q^{ed} - 18(Q^{ed})^2\right)\left(\frac{Q^{ed}}{9\alpha F}\right)}{1 + \frac{Q^{ed}(1-Q^{ed})}{9\alpha^2}} = \frac{\left(\frac{1}{2} - 2Q^{ed} - 18(Q^{ed})^2\right)Q^{ed}}{9\alpha^2 + Q^{ed}(1-Q^{ed})}\frac{\alpha^{ed}}{F}$$

From equation (20) we know that $\frac{dSW^{ed}}{d\alpha} = 0 \Rightarrow Q^{ed}\left(\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right) - 3\alpha^2 = 0$. Using this we get $\frac{d\alpha^{ed}}{dF} = \frac{1 - 4Q^{ed} - 36(Q^{ed})^2}{5 - 8Q^{ed} - 36(Q^{ed})^2}\frac{\alpha^{ed}}{F}$.

(iv) We use this expression for $\frac{d\alpha^{ed}}{dF}$ to determine $\frac{dQ^{ed}(\alpha^{ed})}{dF}$. Now

$\frac{dQ^{ed}(\alpha^{ed})}{dF} = \frac{dQ^{ed}(\alpha^{ed})}{d\alpha^{ed}}\frac{d\alpha^{ed}}{dF} + \frac{dQ^{ed}(\alpha^{ed})}{dF}$. Substituting $\frac{dQ^{ed}(\alpha^{ed})}{d\alpha^{ed}} = \frac{Q^{ed}}{3\alpha^{ed}}$,

$\frac{dQ^{ed}(\alpha^{ed})}{dF} = \frac{Q^{ed}}{3F}$ and the expression for $\frac{da^{ed}}{dF}$ we get

$\frac{dQ^{ed}(\alpha^{ed})}{dF} = \frac{Q^{ed}}{3F}\left(\frac{\left(6-12Q^{ed}-72(Q^{ed})^2\right)}{\left(5-8Q^{ed}-36(Q^{ed})^2\right)}\right)$. Equating $6-8Q^{ed}-48(Q^{ed})^2 = 0$ yields

$Q^{ed} = 0.217$ and equating $5-8Q^{ed}-36(Q^{ed})^2 = 0$ yields $Q^{ed} = 0.278$. Recall, from the first order condition that at α^{ed}, $Q^{ed} < 0.217$. Since both the expressions $6-8Q^{ed}-48(Q^{ed})^2$ and $5-8Q^{ed}-36(Q^{ed})^2$ are decreasing in Q^{ed} and at α^{ed}, $Q^{ed} < 0.217$, hence $\frac{dQ^{ed}(\alpha^{ed})}{dF} > 0$.

Q.E.D.

3.3.3 Socially optimal monitoring rate

The government's objective is to choose a monitoring rate that results in the highest possible quality. From Lemma 5 we know that the least possible quality level is $Q^{ed} = 0.217 - \varepsilon$ for any level of *q*. As discussed above the outcome of the ea-strategy does not depend upon monitoring rate because from Lemma 4 we know that $\alpha^{ea} = 0$ maximizes SW^{ea} and $Q^{ea} = \frac{(1-q)}{2(2-q)}$. Equating this to $Q^{ed} = 0.217 - \varepsilon$ we get $q = \frac{0.132 + 4\varepsilon}{0.566 + 2\varepsilon}$. The result for the socially optimal monitoring rate is summarized in Proposition 3.

Proposition 3. The socially optimal monitoring rate is $\alpha^{ea} = 0$ if $q < \frac{0.132 + 4\varepsilon}{0.566 + 2\varepsilon}$ and the socially optimal quality level is $Q^{ea} = \frac{(1-q)}{2(2-q)}$. In this case, the monopolist chooses the ea-strategy. Otherwise the penalty F should be high enough to generate the monitoring rate α^{ed}

that ensures $Q^{ed} = 0.217 - \varepsilon$ and these two are then the socially optimal monitoring rate and quality level. In this case the monopolist chooses the ed-strategy.

Proof of Proposition 3. Since $Q^{ea} = \frac{(1-q)}{2(2-q)}$ is decreasing in *q*, and if $q < \frac{0.132 + 4\varepsilon}{0.566 + 2\varepsilon}$ then $Q^{ea} > Q^{ed} = 0.217 - \varepsilon$. Therefore in this case $Q^{ea} = \frac{(1-q)}{2(2-q)}$ is the highest possible quality level hence $\alpha^{ea} = 0$ is the socially optimal monitoring rate. If $q \geq \frac{0.132 + 4\varepsilon}{0.566 + 2\varepsilon}$ then $Q^{ea} \leq Q^{ed} = 0.217 - \varepsilon$ and $Q^{ed} = 0.217 - \varepsilon$ is the socially optimal quality. In this case the government needs to ensure that its policy generates this quality level which requires a penalty level that will result in the monitoring rate α^{ed} to generate $Q^{ed} = 0.217 - \varepsilon$. Recall from Lemma 1 that at $\hat{\alpha}$, $Q^{ed}(\hat{\alpha}) < Q^{ea}$. Since at α^{ed}, $Q^{ea} \leq Q^{ed}$ it means that $\alpha^{ed} > \hat{\alpha}$ and we have shown that $\alpha^{ed} < \alpha_{\max}$. This means that α^{ed} satisfies the range $\alpha \in [\hat{\alpha}, \alpha_{\max}]$ over which the ed-strategy is weakly dominant and the relevant social welfare is SW^{ed}. So the monopolist chooses the ed-strategy in this case. Q.E.D

Appendix

Proof of Lemma 1. (i) The follows from the fact that π_m^{ea} is independent of α and π_m^{ed} is increasing in α in the range $\alpha \in [0, \alpha_{\max}]$. Thus the single crossing property is satisfied. This is shown in Figure 1. (ii) Suppose at $\hat{\alpha}$, $Q^{ed}(\hat{\alpha}) = Q^{ea} = \frac{1-q}{2(2-q)}$. Substituting this in π_m^{ed} as given in equation (9) we get $\pi_m^{ed} = \frac{3(1-q)^2}{8(2-q)^2} - \frac{4(1-q)^3}{8(2-q)^3}$. Now $\pi_m^{ed} - \pi_m^{ea} = \frac{q(1-q)^2}{4(2-q)^3} > 0$. But at $\hat{\alpha}$, $\pi_m^{ed}(\hat{\alpha}) = \pi_m^{ea}$ which means $Q^{ed}(\hat{\alpha}) \neq Q^{ea} = \frac{1-q}{2(2-q)}$. Further since $\frac{dQ^{ed}}{d\alpha} > 0$ it means that the equality $Q^{ed}(\alpha) = Q^{ea} = \dfrac{1-q}{2(2-q)}$ holds only for $\alpha > \hat{\alpha}$. Thus $Q^{ed}(\hat{\alpha}) < Q^{ea}$.

Q.E.D.

Proof of Lemma 5. (i) The first order condition requires

$\dfrac{dSW^{ed}}{d\alpha} = \left(\dfrac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right)\dfrac{dQ^{ed}}{d\alpha} - \alpha = 0$. This means that at the optimum it must be the case that $\left(\dfrac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right) > 0$ because $\dfrac{dQ^{ed}}{d\alpha} > 0$. The second derivative is

$\dfrac{d^2SW^{ed}}{d\alpha^2} = \left(\dfrac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right)\dfrac{d^2Q^{ed}}{d\alpha^2} + \left(-1-12Q^{ed}\right)\left(\dfrac{dQ^{ed}}{d\alpha}\right)^2 - 1$. The first term is negative because at the optimum $\left(\dfrac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right) > 0$ and we know that $\dfrac{d^2Q^{ed}}{d\alpha^2} < 0$. The second term is negative which is evident from observation. Thus $\dfrac{d^2SW^{ed}}{d\alpha^2} < 0$ which means that the second order condition for maximization is satisfied. Let us evaluate the sign of $\dfrac{dSW^{ed}}{d\alpha}$ at $\alpha = \alpha_{\max}$ where $Q^{ed} = \dfrac{1}{4}$. Now $\dfrac{dSW^{ed}}{d\alpha}\Big|_{\alpha=\alpha_{\max}} < 0$ because at $\alpha = \alpha_{\max}$, $\left(\dfrac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right) = -\dfrac{1}{8} < 0$ and $\dfrac{dQ^{ed}}{d\alpha} > 0$. This shows that $\alpha^{ed} < \alpha_{\max}$.

(ii) Total differentiating $\dfrac{dSW^{ed}}{d\alpha} = \left(\dfrac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right)\dfrac{dQ^{ed}}{d\alpha} - \alpha = 0$ with respect to *F* yields

$\dfrac{\delta^2SW^{ed}}{\delta(\alpha^{ed})^2}d\alpha^{ed} + \dfrac{\delta^2SW^{ed}}{\delta F\delta\alpha^{ed}}dF = \dfrac{\delta^2SW^{ed}}{\delta(\alpha^{ed})^2}d\alpha^{ed} + \left(-1-12Q^{ed}\right)\dfrac{\delta Q^{ed}}{\delta F}dF = 0$. Since $\dfrac{\delta^2SW^{ed}}{\delta(\alpha^{ed})^2} < 0$ and $\left(-1-12Q^{ed}\right)\dfrac{\delta Q^{ed}}{\delta F} < 0$ because $\dfrac{\delta Q^{ed}}{\delta F} > 0$, therefore, $\dfrac{\delta\alpha^{ed}}{\delta F} < 0$. Total differentiating $\pi_m^{ed}(\hat{\alpha}, F) = \pi_m^{ea}$ with respect to $\hat{\alpha}$ and *F* we get $3Q^{ed}\left(1-4Q^{ed}\right)\left(\dfrac{\delta Q^{ed}}{\delta F}dF + \dfrac{\delta Q^{ed}}{\delta\hat{\alpha}}d\hat{\alpha}\right) = 0$.

Since $\frac{\delta Q^{ed}}{\delta F} > 0$ and $\frac{dQ^{ed}}{d\alpha} > 0$, and $\alpha \leq \alpha_{\max}$ which implies that $Q^{ed} \leq \frac{1}{4}$, therefore,

$\frac{\delta\hat{\alpha}}{\delta F} = -\frac{\hat{\alpha}}{F} < 0$. (iii) Differentiating $SW^{ed}(\alpha^{ed})$ as given in equation (14) with respect to *F*

gives us $\frac{dSW^{ed}(\alpha^{ed})}{dF} = \frac{dSW^{ed}}{d\alpha^{ed}}\frac{d\alpha^{ed}}{dF} + \frac{dSW^{ed}}{dF} = \frac{dSW^{ed}}{dF} = \left(\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right)\frac{dQ^{ed}}{dF} > 0$

because at α^{ed}, $\frac{dSW^{ed}}{d\alpha^{ed}} = 0$ and $\left(\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2\right) > 0$ as shown in the proof of Lemma 5

(i), and $\frac{dQ^{ed}}{dF} > 0$. This also shows that the peak of a lower SW^{ed} curve associated with a lower *F* is contained within the peak of a higher SW^{ed} curve associated with a higher *F*. Further the two SW^{ed} curves intersect at the downward sloping segments. This we get by solving for $\frac{1}{2} - Q^{ed} - 6(Q^{ed})^2 = 0$ which yields $Q^{ed} = 0.217$. Thus we can say that for

$Q^{ed} < 0.217$ $\frac{dSW^{ed}}{dF} > 0$ and for $Q^{ed} > 0.217$ $\frac{dSW^{ed}}{dF} < 0$. Now $\frac{dQ^{ed}}{dF} = \frac{(1-q)^{\frac{1}{3}}\hat{\alpha}^{\frac{1}{3}}F^{-\frac{2}{3}}}{3q^{\frac{1}{3}}}$,

$\frac{dQ^{ed}}{d\hat{\alpha}} = \frac{(1-q)^{\frac{1}{3}}\hat{\alpha}^{-\frac{2}{3}}F^{\frac{1}{3}}}{3q^{\frac{1}{3}}}$, $\frac{\delta\hat{\alpha}}{\delta F} = -\frac{\hat{\alpha}}{F}$, and

$\frac{dQ^{ed}}{d\hat{\alpha}}\frac{d\hat{\alpha}}{dF} = -\frac{(1-q)^{\frac{1}{3}}\hat{\alpha}^{-\frac{2}{3}}F^{\frac{1}{3}}}{3q^{\frac{1}{3}}}\frac{\hat{\alpha}}{F} = -\frac{(1-q)^{\frac{1}{3}}\hat{\alpha}^{\frac{1}{3}}F^{-\frac{2}{3}}}{3q^{\frac{1}{3}}} = -\frac{dQ^{ed}}{dF}$. Hence, $\frac{dSW^{ed}(\hat{\alpha})}{dF} = 0$.

(iv) $\frac{dSW^{ed}(\hat{\alpha})}{dF} = 0$ and $\frac{\delta\hat{\alpha}}{\delta F} = -\frac{\hat{\alpha}}{F} < 0$ implies that for any increase in *F*, $\hat{\alpha}$ decreases horizontally in the *SW*-α plane. $\frac{dSW^{ed}(\alpha^{ed})}{dF} > 0$ and $\frac{\delta\alpha^{ed}}{\delta F} < 0$ implies that an increase in *F* reduces α^{ed} but $SW^{ed}(\alpha^{ed})$ increases, hence the reduction in α^{ed} is not horizontal. This means that the reduction in $\hat{\alpha}$ is greater than the reduction in α^{ed} for a given increase in *F*.

Now suppose that $\hat{\alpha}$ is to the right of α^{ed}. Consider increases in *F*. Since the fall in $\hat{\alpha}$ is greater than the fall in α^{ed}, there will be a critical F at which $\hat{\alpha} = \alpha^{ed}$. This means at that critical *F* $SW^{ed}(\hat{\alpha}) = SW^{ed}(\alpha^{ed})$. This is not possible because $\frac{dSW^{ed}(\hat{\alpha})}{dF} = 0$, $\frac{dSW^{ed}(\alpha^{ed})}{dF} > 0$ and $SW^{ed}(\hat{\alpha}) < SW^{ed}(\alpha^{ed})$ since α^{ed} maximizes SW^{ed}. Thus $\hat{\alpha}$ is to the left of α^{ed} that is, $\hat{\alpha} < \alpha^{ed}$ which means that α^{ed}, with $\alpha^{ed} \in (\hat{\alpha}, \alpha_{\max})$, maximizes SW^{ed}.

Q.E.D.