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Abstract

We explore whether the nature of piracy or the counterfeiting activity and the competition between the copyright holder and the pirate(s) matter in a given regime of Intellectual Property Right (IPR) protection. Generally, the nature of piracy can be of two types, commercial and end-user; and the nature of competition between copyright holder and if the pirate is commercial can be either in price or quantity depending on the pirated good. We find irrespective of the nature of piracy or competition, when the consumers' tastes are sufficiently diverse and IPR protection is weak, it is profitable for the copyright holder to accommodate the pirate(s), while deter the pirate(s) in all other situations. The relationship between the quality of pirated good and piracy rate can be monotonic or non-monotonic. Piracy is more likely to survive under commercial piracy than under end-user piracy. The relationship between private and public anti-piracy measures is non-monotonic.

Keywords: IPR protection, private copyright protection, piracy rate, product quality, commercial piracy, end-user piracy

JEL Classifications: D23, D43, L13, L86, O3

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1. Introduction

Recently, Lu and Poddar (2012) analyzed the problem of copyright violations where the copyright holder makes costly investment to deter a commercial pirate in a given regime of intellectual property rights (IPR) protection. A two-stage game is considered where in the first stage the copyright holder makes costly investment to raise the cost of piracy of the commercial pirate. In the second stage, if the pirate survives, it competes with the copyright holder in prices by producing an inferior pirated good. Depending on the strength of IPR protection regime and consumer taste heterogeneity, the copyright holder can choose to accommodate or deter the pirate. The main result is, when the consumers' tastes are sufficiently diverse and the IPR protection is weak, it is profitable for the copyright holder to accommodate the pirate and in all other cases, it is profitable to deter. Several interesting comparative statics analyses are done following the result.

In this paper, we extend the above study in two directions. Firstly, we change the nature of competition between the copyright holder and the commercial pirate by bringing competition in quantities as opposed to prices. While the price competition between the commercial pirate and copyright holder is appropriate for digital products, however, when it comes to non-digital goods like counterfeit garments, clothing, shoes, accessories etc. (i.e. counterfeiting activity in fashion and related goods industries), the competition between the commercial pirate and the original producer is more of a quantity competition. There the pirate wants to maximize the sale volume through a brisk business by (ab)using the brand name of the original good to those consumers who are not very certain about the authenticity of the good.[1] The second extension is in the line of nature of piracy. Instead of commercial piracy, we explore when the pirates are end-users. This is relevant in the context of digital products where end-users pirate the product for personal consumption.[2]

We are interested to see how the results get affected in these two new contexts of piracy. We also do several comparative analysis to gain more insights on how the nature of piracy and competition matters to the copyright holder, the users of the product i.e. the

[1] There may be concern of assuming one commercial pirate instead of many under commercial piracy. We view this problem in the following way. The rationale for considering a single competing pirate is the fact that typically there is a local syndicate that manufactures pirated goods, and distributes them through different small retail channels. The single commercial pirate in our paper replicates the local syndicate.

[2] Most common digital products are movies, music, or various software applications.

consumers, as well as the IPR protection authority.

In the literature, both commercial and end-user piracy are studied in detail. Studies by Conner and Rumelt (1991), Takeyama (1994), Slive and Bernhardt (1998), Shy and Thisse (1999), Chen and Png (2003), King and Lamp (2003), Bae and Choi (2006), Belleflame and Picard (2007) among others mainly focused on end-user piracy; whereas studies by Banerjee (2003, 2011, 2013), Lu and Poddar (2012), Kiema (2008), Jaisingh (2009), Martinez-Sanchez (2010) among others focused on commercial piracy.[3] However, to the best of our knowledge no comparative study between commercial and end-user piracy has been done in detail so far. We also aim to fill that gap in the literature.[4]

Like in Lu and Poddar (2012), in our commercial piracy model, we assume there is one original product developer (i.e. the copyright holder), a commercial pirate and a group of heterogeneous consumers. In the end-user piracy model, we assume there is one original product developer and a group of heterogeneous consumers who are also the potential pirates. In both models of piracy, the original product developer makes costly investment to deter piracy in a given regime of IPR protection. We call the copyright holder's effort to deter piracy as private measure for anti-piracy and the IPR protections as the public measure of anti-piracy. The IPR protection level is assumed to be exogenous in the models and treated as a parameter.[5] Under commercial piracy, the original producer targets the commercial pirate and invests to raise the cost of piracy of the commercial pirate. We assume the pirate produces a similar product but of lower quality.[6] The copyright holder

[3] For a comprehensive survey on the recent developments on the theory of digital piracy see Belleflamme and Peitz (2012) and for empirical analysis see Waldfogel (2012).

[4] Piracy in the developed world is relatively less, however if some piracy exists it is more end-user type piracy rather than commercial. Whereas in the poorer countries commercial piracy is more pronounced than end-user piracy. Apart from the consumers' wealth and purchasing capacities, the difference in IPR laws and enforcements are the reasons for the difference in piracy situations in developed and less developed world.

[5] It is fairly well documented that different countries have different levels of IPR protections; it can be weak or strong. More importantly, for a country it takes a long time to adjust its IPR policy (more so if the government of that country is not very pro-active to reform IPR related policies), hence we assume it to be exogenous in our model. We do acknowledge that in many studies, IPR policy instruments, like monitoring the pirate and imposing penalty, are modeled endogenously, however in this analysis since our focus is different we do not take that route. Nevertheless, we will do comparative statics analysis on various levels of IPR measures in the analysis.

[6] This assumption is standard in the literature of piracy. For digital piracy, even if the digital copy is identical to the original one, one would not get a guarantee or any follow up service associated with the product if buys a pirated version.

may choose strategically to completely deter or accommodate the entry of the pirate. If the original producer finds it unprofitable to deter piracy completely, it chooses to accommodate the pirate. If entry is accommodated, we assume the original producer and the pirate compete in the product market in quantities. In the model of end-user piracy, the original product developer is a monopolist, but faces numerous potential end-user pirates who are willing to pirate the product instead of buying it. The IPR protection policy as well as the deterrence efforts of the original producer now targets the end-users in a similar way to limit piracy.

In characterizing the equilibrium outcomes, we find a very generic result that also reaffirms the result of Lu and Poddar (2012). When the buyers' or consumers' tastes are sufficiently diverse and IPR protection is weak, it is profitable for the original producer to accommodate the pirate(s), while deter the pirate(s) in all other situations. Thus, the result is true irrespective of the nature of competition (price or quantity) between the copyright holder and the commercial pirate; and of the nature of piracy (commercial or end-user).

We are also interested to know how the quality of the pirated good impacts the piracy rate as in reality the quality or reliability of the pirated product could vary widely from very unreliable to highly reliable in comparison to the original product. The quality of the pirate good depends on the copying technology and other factors, like consumers' valuation or perception of the pirated good and antipiracy polices. Now to find the relationship between the quality of the pirated good and the piracy rate, we find both monotonic and non-monotonic relationship may exist. In our framework, the relationship between the rate of piracy and the quality of the pirated product is monotonically increasing under commercial piracy whereas the relationship is non-monotonic under end-user piracy.[7] Further, as for the relationship between the optimal deterrence level from the copyright holder and the quality of the pirated good, we find under commercial piracy, the optimal deterrence level decreases as the pirated quality increases under accommodation, and the reverse is true under deterrence. Under end-user piracy, the optimal deterrence level increases as the pirated quality increases under accommodation and it also increases

[7] Lopez-Cunat and Martina-Sanchez (2013) in their model of commercial piracy also found a similar non-monotonicity. Non-monotonicity is also observed under price competition (Lu and Poddar 2012).

with the pirated quality under deterrence except when the consumers' tastes are sufficiently diverse and the pirated quality is sufficiently high.

In general, piracy is more likely to survive under commercial piracy (irrespective of price and quantity competition) than under end-user piracy for any given quality of the pirated product and for a given level of IPR protection. As far as the optimal deterrence effort of the copyright holder is concerned, we find that to completely deter piracy, the original producer has to give higher level of effort (hence higher piracy deterring investment) under quantity competition whereas interestingly we find the same level of deterrence effort is needed under price competition or end-user piracy in a comparable situation. When the pirate is accommodated, the original producer has to give the maximum effort for deterrence under end-user piracy and least effort under commercial piracy with quantity competition. These results are consequences of the fact that the copyright holder generally faces a lower degree of competition under commercial piracy compared to the end-user piracy when other factors are kept constant.

Finally, to find the interaction between two anti-piracy measures i.e. public protection (IPR protection) and private protection (optimal deterrence level from copyright holder), we see when piracy is accommodated, the two anti-piracy measures are always complements to each other; while they are always substitutes when piracy is deterred under both modes of piracy (commercial and end-user). Thus, the overall relationship between private and public anti-piracy measures is non-monotonic for both types of piracy which also has policy implications.

Our analysis here encompasses both the digital and non-digital piracy. In that sense, this study can also be considered to be a general study on the implications of copyright violations. We believe the findings in our comparative statics analysis are empirically testable. For example, it is important to verify whether there indeed exists a monotonic or non-monotonic relationship between the optimal level of deterrence by the copyright holder and the strength of IPR protection in the economy; or whether we can find a monotonic or non-monotonic relationship between the perceived quality of the pirated product and the prevailing rate of piracy in the economy.

The rest of the paper is organized as follows. In the next section, we lay out the basic framework of commercial piracy. In section 3, we do our analysis of commercial piracy. In section 4, we do the analysis for end-user piracy. Section 5 compares findings under commercial and end-user piracy. Section 6 concludes.

2. The Model of Commercial Piracy

2.1 The Original Firm and the Pirate

Consider an original firm and a commercial pirate. The pirate has the know-how or the technology to copy/counterfeit the original product. We assume the pirate produces counterfeit items, which are of lower quality than the original. The product quality of the pirated good (compared to original) is captured by the parameter q, $q \in (0,1)$.[8]

We consider a two-period model, where in the first period $(t=1)$, the original product developer undertakes costly investment in order to deter or limit piracy. It adopts the following entry deterring strategy. It tries to deter the pirate by increasing the cost of counterfeiting, in particular, raising the marginal cost of producing a fake item. The potential pirate appears in the market of the original product in the second time period $(t=2)$. We assume the higher the entry deterring investment made by the original product developer in the first period (the higher the deterrence level), the higher would be the cost of counterfeiting for the pirate. The pirate if survives, competes with the original developer in quantities.

We assume at $t=1$, the cost of investment of the original product developer to choose the level of deterrence, x, is given by $c_o(x) = x^2/2$. Thus, if the profit of the product developer at $t=2$ is denoted by π_o^2 then the net profit of the developer at the end of the game is $\pi_o = \pi_o^2 - c_o(x) = \pi_o^2 - x^2/2$. When the level of deterrence is x, the marginal cost of production for the pirate will be $c+x$, where c is a parameter $(c>0)$ *exogenously* given. If the pirate is in the market at *t* = *2* then its profit function $\pi_p = (\text{Re}venue) - (c+x)q_p$ where q_p is the quantity sold by the pirate. We would like to interpret c as the degree or the strength of IPR protection exogenously given. It essentially captures the strength of legal protection and enforcement to stop or limit piracy and it is

[8] Implicitly we assume the quality index of the original good is 1.

beyond the control of the original firm (i.e. the copyright holder). It is generally understood that the government or the regulatory authority can influence c.[9] In our model, we interpret c as the public effort from the government and x as the private effort from the product developer to stop/limit piracy. We assumed an additive form between c and the level of deterrence x that is chosen by the original firm. The reason is as follows. We view the pirate's copying cost has two components. One is due to original producer's private effort to deter piracy, which may include technological adoption to protect counterfeiting; and/or it could be private monitoring, identifying and suing the pirate and all of these efforts can be reflected in x. The other component is due to the IPR protection regime i.e. the strength of IPR legislations and enforcements which is reflected in c. Both the original firm's private effort (investment) and the legal protection and enforcement of copyright legislations (public protection) contribute to the deterrence of piracy.

2.2 Consumers' Preferences

Consider a continuum of consumers indexed by $X \in [0, \theta]$. X measures the taste or the consumer's willingness to pay for the product. A high value of X means high valuation for the product and low value of X means low valuation for the product. Therefore, one consumer differs from another on the basis of his/her valuation or the taste for the particular product. Valuations are uniformly with density $1/\theta$ distributed over the interval $[0, \theta]$.[10] Each consumer purchases at most one unit of the good. A type-X consumer's utility function is given as:[11]

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[9] According to a recent study by Andres (2006) (also see Park and Ginarte (1997)), the strength of IPR protection of a country mainly consists of two categories: *membership in the international copyright treaties* and *enforcement provisions*. Going by the definition and measure of the strength of IPR protection as discussed in Andres (2006), we can generally find a relatively high c in the developed countries where piracy is taken as a serious crime; hence it raises the cost of piracy significantly. On the contrary, in most of the developing countries, we will probably find c to be relatively low, because even if the laws are there to stop piracy, the enforcement policies against piracy may not be as strict; hence cost of piracy would remain relatively small.

[10] So the number of consumers is normalized to one.

[11] The utility representation is borrowed from the standard model of vertical product differentiation in the literature (see Tirole (1988)).

$$U = \begin{cases} X - p_o & \text{if buys original product,} \\ qX - p_p & \text{if buys pirated product,} \\ 0 & \text{if buys none,} \end{cases}$$

where p_o and p_p are the prices of the original and pirated products respectively.[12]

2.3 Deriving Demands of the Product Developer and the Pirate

The demand for the original product and for the pirated product, D_o and D_p, can be derived from the distribution of buyers as follows.

Recall that consumers are heterogeneous with respect to their values towards the product. Thus, the marginal consumer, X who is indifferent between buying the original product and the pirated version, is given by $X - p_o = qX - p_p$, or $X = \frac{p_o - p_p}{1-q}$. The marginal consumer, Y who is indifferent between buying the pirated product and not buying any product, is given by $qY - p_p = 0$, or $Y = \frac{p_p}{q}$. Thus, the demand for original product is $D_o = \frac{1}{\theta}\int_X^{\theta} dx = \left[(1-q)\theta - (p_o - p_p)\right] / (1-q)\theta$ and the demand for pirated product is $D_p = \frac{1}{\theta}\int_Y^{X} dx = (qp_o - p_p)/q(1-q)\theta$.

Note that we have implicitly assumed that $qp_o \geq p_p$ when we derive the demand functions as above. When this assumption does not hold true, the demand for pirated product becomes zero while the demand for original producer is $D_o = (\theta - p_o)/\theta$. Thus, we write the demand functions as the following:

$$D_o = \begin{cases} \left[(1-q)\theta - (p_o - p_p)\right] / (1-q)\theta & \text{if } qp_o \geq p_p \\ (\theta - p_o)/\theta & \text{otherwise} \end{cases}, \tag{1}$$

[12] Note that $q = 0$ will eliminate the pirated product, while $q = 1$ will make the two products identical. In our model $q = 1$ is never possible as we have assumed that the pirated good is of lower quality. Also technically, $q \in (0,1)$ is needed so that demands, prices and profits are not indeterminate.

and

$$D_p = \begin{cases} \left(qp_o - p_p\right)/q(1-q)\theta & \text{if } qp_o \geq p_p \\ 0 & \text{otherwise} \end{cases}. \tag{2}$$

Assume both the original developer and the pirate have positive demand, then inverting the demand system in (1) and (2), one can obtain the following inverse demand functions:

$$p_o = \theta\left(1 - D_o - qD_p\right), \tag{3}$$

$$p_p = q\theta\left(1 - D_o - D_p\right) . \tag{4}$$

3. Analysis and Main Results: Commercial Piracy

In what follows, we first focus on what would be the best piracy-deterring strategy *x* (hence, the optimal piracy deterring private investment in response to potential piracy) for the original product developer given an enforcement environment of IPR protection (i.e. given *c*). We look for subgame perfect equilibrium of the two-period game and solve for it using backward induction. We first obtain equilibrium quantities in the quantity competition stage in the second period, and then work out the choice of the optimal level of deterrence by the original firm in the first period. Note that the original producer can decide to accommodate or deter entry of the pirate completely.

3.1 The Entry Accommodation Equilibrium and Entry Deterrence Equilibrium

In the second stage of the game, the original developer chooses D_o to maximize $\pi_o^2\left(D_o, D_p\right) = \theta\left(1 - D_o - qD_p\right)D_o$, while the pirate chooses D_p to maximize $\pi_p\left(D_o, D_p\right) = \left[q\theta\left(1 - D_o - D_p\right) - c - x\right]D_p$. From the first-order conditions for profit maximization, we can obtain both firms' reaction functions:

$$D_o = \frac{1}{2}\left(1 - qD_p\right),$$

$$D_p = \frac{1}{2}\left(1 - D_o - \frac{c+x}{q\theta}\right).$$

The equilibrium quantities are then

$$D_o = \frac{1}{(4-q)\theta}\left(c + x + (2-q)\theta\right),$$

$$D_p = \frac{1}{q(4-q)\theta}\left(q\theta - 2(c+x)\right).$$

Note that only when $2(c+x) < q\theta$, $D_p > 0$. So if the original producer chooses *x* such that $2(c+x) \geq q\theta$, i.e., $x \geq q\theta/2 - c$, then $D_p = 0$. It is also clear that if $c \geq q\theta/2$, there is no need to deter piracy.

When $2(c+x) < q\theta$, one can then obtain the following equilibrium prices and profits for both firms:

$$p_o = \frac{1}{4-q}\left(c + x + (2-q)\theta\right),$$

$$p_p = \frac{(2-q)(c+x) + q\theta}{4-q},$$

$$\pi_o^2 = \frac{1}{(4-q)^2\theta}\left(c + x + (2-q)\theta\right)^2,$$

$$\pi_p = \frac{1}{q(4-q)^2\theta}\left(q\theta - 2(c+x)\right)^2.$$

Note that $\pi_o^2 = \theta/4$ when $x = q\theta/2 - c$, which is the same as when the firm chooses a deterrence level higher than $q\theta/2 - c$.[13] Thus, when the deterrence cost is taken into account, $x > q\theta/2 - c$ is strictly dominated by $x = q\theta/2 - c$.

In stage 1, the original developer chooses the deterrence level *x* to maximize $\pi_o = \pi_o^2 - \frac{x^2}{2} = \frac{1}{(4-q)^2\theta}\left(c + x + (2-q)\theta\right)^2 - \frac{x^2}{2}$. To find the optimal deterrence level *x*, we first find $\frac{d\pi_o}{dx} = \frac{2\left(c + x + (2-q)\theta\right)}{(4-q)^2\theta} - x$ and $\frac{d^2\pi_o}{dx^2} = \frac{2}{(4-q)^2\theta} - 1$. Note that when

[13] When $x > q\theta/2 - c$, the original producer, as a monopolist, will choose $p_o = \theta/2$ and obtain profits of $\theta/4$.

evaluated at x=0, $\frac{d\pi_o}{dx}=\frac{2\left(c+\left(2-q\right)\theta\right)}{\left(4-q\right)^2\theta}>0$. We then distinguish two cases depending on whether $d^2\pi_o/dx^2$ is positive or negative.

When $\left(4-q\right)^2\theta\le 2$, $\frac{d^2\pi_o}{dx^2}\ge 0$. Since $\frac{d\pi_o}{dx}>0$ when evaluated at x=0, we must have $\frac{d\pi_o}{dx}>0$ for all $x\ge 0$, i.e., the profit function is strictly increasing in x. The original producer will choose a deterrence level x as big as possible subject to the constraint $2\left(c+x\right)\le q\theta$. Thus, the optimal deterrence level is $x^*=q\theta/2-c$.

When $\left(4-q\right)^2\theta>2$, $\frac{d^2\pi_o}{dx^2}<0$. The profit function is concave in x. When evaluated at $x=\frac{q\theta}{2}-c$, $\frac{d\pi_o}{dx}=c-\frac{q\left(4-q\right)\theta-2}{2\left(4-q\right)}$, which is positive when $c>\frac{q\left(4-q\right)\theta-2}{2\left(4-q\right)}$ and negative when $c<\frac{q\left(4-q\right)\theta-2}{2\left(4-q\right)}$. Therefore, when $c>\frac{q\left(4-q\right)\theta-2}{2\left(4-q\right)}$, the optimal deterrence level is $x^*=q\theta/2-c$, while when $c<\frac{q\left(4-q\right)\theta-2}{2\left(4-q\right)}$, the optimal deterrence level is determined by $\frac{d\pi_o}{dx}=\frac{2\left(c+x+\left(2-q\right)\theta\right)}{\left(4-q\right)^2\theta}-x=0$ and therefore,

$$x^*=\frac{2\left(c+\left(2-q\right)\theta\right)}{\left(4-q\right)^2\theta-2}.$$

We thus have the following proposition characterizing the entry accommodation equilibrium and entry deterrence equilibrium.

Define $\phi\left(q,\theta\right)\equiv\frac{q\left(4-q\right)\theta-2}{2\left(4-q\right)}$.

Proposition 1

(i) *When* $\theta \le \frac{2}{q(4-q)}$ *and* $c < q\theta/2$, *the original producer's optimal level of deterrence is* $x^* = q\theta/2 - c$. *In this case, it deters the pirate and the pirate has no demand.*

(ii) *When* $\theta > \frac{2}{q(4-q)}$ *and* $c < q\theta/2$,

(a) when $0 \le c \le \phi(q,\theta)$, *the original producer's optimal level of deterrence is* $x^* = 2\left(c + (2-q)\theta\right) \Big/ \left((4-q)^2\theta - 2\right)$. *In this case, it accommodates the pirate and shares the market with the pirate.*

(b) When $\phi(q,\theta) \le c < q\theta/2$, *the original producer's optimal level of deterrence is* $x^* = q\theta/2 - c$. *In this case, it deters the pirate and the pirate has no demand.*

(iii) *When* $c \ge q\theta/2$, *there is no need to deter the pirate strategically. Piracy is blockaded anyway due to exogenous high level of IPR protection.*

The condition $\theta \le \frac{2}{q(4-q)}$ in Proposition 1(*i*) can be interpreted as when the consumers' tastes are not sufficiently diverse. In such a case, the original producer deters the pirate even when the degree of intellectual property right (IPR) protection is not sufficiently high (i.e. $c < q\theta/2$) as the market is not diversified enough to profitably share with the pirate.

On the contrary, when the consumers' tastes are sufficiently diverse (i.e. $\theta > \frac{2}{q(4-q)}$ i.e. the market is diversified enough), the sharing possibility with the pirate increases, however, relatively high degree of IPR protection (i.e. $\phi(q,\theta) \le c < q\theta/2$) helps the original producer to deter the pirate in this situation. On the other hand, in this case (i.e. when the market is diversified enough), complete deterrence is too costly if the degree of intellectual property right is low (i.e. $c \le \phi(q,\theta)$), there the original producer accommodates the pirate. Note that a necessary condition for entry accommodation to be optimal is $\theta > 2/3$ since $q(4-q)$ is maximized on the interval [0,1] at q=1 and the maximum is 3.

3.2 Comparative Statics

Denote the equilibrium prices we obtained before the optimal deterrence effort is plugged into the relevant expressions as functions of q and x, namely, $p_o(q,x)=\frac{1}{4-q}\left(c+x+(2-q)\theta\right)$, $p_p(q,x)=\frac{(2-q)(c+x)+q\theta}{4-q}$.

Observation: $\frac{\partial p_o(q,x)}{\partial q}<0$ and $\frac{\partial p_p(q,x)}{\partial q}>0$ (proofs in Appendix 1).

Denote the equilibrium prices after the optimal deterrence effort is plugged into $p_o(q,x)$ and $p_p(q,x)$ as functions of q, namely,

$$p_o(q)=\frac{1}{4-q}\left(c+\frac{2\left(c+(2-q)\theta\right)}{(4-q)^2\theta-2}+(2-q)\theta\right)=\frac{\theta(4-q)\left(c+(2-q)\theta\right)}{(4-q)^2\theta-2},$$

$$p_p(q)=\frac{(2-q)\left(c+\frac{2\left(c+(2-q)\theta\right)}{(4-q)^2\theta-2}\right)+q\theta}{4-q}=\theta\frac{c(2-q)(4-q)+q(4-q)\theta+2(1-q)}{(4-q)^2\theta-2}.$$

Lemma 1: $\frac{dp_o(q)}{dq}<0$ and $\frac{dp_p(q)}{dq}>0$ (proofs in Appendix 1).

3.2.1 Optimal level of deterrence (*x*) and quality of the pirated product (*q*)

Lemma 2: *Under commercial piracy, for entry accommodation we have* $\frac{\partial x^*}{\partial q}<0$ *and for entry deterrence* $\frac{\partial x^*}{\partial q}>0$.

Proof: Under accommodation, $x^* = 2\left(c+(2-q)\theta\right)\Big/\left((4-q)^2\theta-2\right)$.

$$\frac{\partial x^*}{\partial q} = \frac{-2\theta}{\left((4-q)^2\theta-2\right)^2}\left(q(4-q)\theta-2-2(4-q)c\right)<0 \text{ since } c\le\phi(q,\theta)\equiv\frac{q(4-q)\theta-2}{2(4-q)}.$$

Under deterrence, $x^* = q\theta/2-c$. Clearly, $\frac{\partial x^*}{\partial q}>0$. □

It shows that as the pirated product becomes more reliable, the original producer will increase investment on piracy deterrence as long as it can keep the pirate out of the market and can serve the market alone. However, when it fails to completely deter the pirate and has to share the market, it is optimal to reduce the investment on deterrence. The idea is that the copyright holder tries its best to keep the pirate out, however if it is not possible then increasing the piracy deterrence investment is not optimal.

To illustrate Proposition 1 and Lemma 2, we provide two numerical examples. In the first example, we fix $\theta=1$ and $c=0.1$. Then it is straightforward to obtain the following optimal deterrence effort as a function of *q*:

$$x^* = \begin{cases} 0 & \text{if } 0<q\le 0.2, \\ q/2-0.1 & \text{if } 0.2\le q\le 0.83114, \\ 2(2.1-q)\Big/\left((4-q)^2-2\right) & \text{if } 0.83114\le q<1. \end{cases}$$

We see that for a very low quality (*q*) of the pirated good, IPR protection (*c*) is sufficient to stop piracy. In other words, when the pirated good is of very low quality, the commercial pirate is not a threat to the copyright holder and no private deterrence is necessary to stop piracy. It becomes a real threat to the copyright holder when *q* crosses a certain threshold i.e. becomes higher in quality. Then the copyright holder has to invest in deterrence and it is only successful to deter the pirate completely when the pirated quality stays in the intermediate (medium) range. However, accommodating the pirate is profitable to the copyright holder when the quality of the pirated good is high. Further we note that under accommodation, $\frac{dx(q)}{dq}=\frac{d}{dq}\left(\frac{2(2.1-q)}{(4-q)^2-2}\right)=\frac{5q^2-21q+14}{5\left((4-q)^2-2\right)^2}<0$ for $0.83114<q<1$;

while under complete deterrence, $\frac{dx(q)}{dq}=\frac{d}{dq}\left(\frac{q}{2}-0.1\right)=\frac{1}{2}>0$.

In the second example, we fix $\theta=0.5$ and $c=0.1$. Then we can get the following optimal deterrence effort as a function of *q*:

$$x^*=\begin{cases}0 & \text{if } 0<q\le 0.4,\\ q/4-0.1 & \text{if } 0.4\le q<1.\end{cases}$$

In this example, the consumers' tastes are not sufficiently diverse, accommodation is never optimal for the original producer. Also under complete deterrence, $\frac{dx(q)}{dq}=\frac{d}{dq}\left(\frac{q}{4}-0.1\right)=\frac{1}{4}>0$.

3.3 Rate of Piracy and Quality of the Pirated Product

We define the ratio of $D_p/(D_o+D_p)$ to measure the rate of piracy. Thus the higher the ratio, the higher will be the rate of piracy. When $\theta>\frac{2}{q(4-q)}$ and $c\le\phi(q,\theta)$ (equivalently, $\frac{2\theta+c-\sqrt{4\theta^2+c^2-4\theta c-2\theta}}{\theta}\le q<1$), i.e. when the original firm accommodates the pirate, it is straightforward to get $\frac{D_p}{D_o+D_p}=\frac{(4-q)(q\theta-2c)-2}{(4-q)(q(3-q)\theta-(2-q)c)-2}$. In all the other cases, entry is either deterred or blockaded; thus, the rate of piracy is zero.

When $\theta>\frac{2}{q(4-q)}$ and $c\le\phi(q,\theta)$, straightforward computation yields

$$\frac{\partial}{\partial q}\left(\frac{D_p}{D_o+D_p}\right)=\frac{2(4-q)^2c^2+4\left((4-q)^2(1-q)\theta+2-q\right)c+\left(q^2(4-q)^2\theta+2\left(8-12q+3q^2\right)\right)\theta}{\left((4-q)\left(q(3-q)\theta-(2-q)c\right)-2\right)^2}. \quad (5)$$

Since $\theta > \dfrac{2}{q(4-q)}$ and thus

$q^2(4-q)^2\theta + 2(8-12q+3q^2) > 2q(4-q) + 2(8-12q+3q^2) = 4(2-q)^2$, the last term in the numerator is positive, $\dfrac{\partial}{\partial q}\left(\dfrac{D_p}{D_o + D_p}\right) > 0$. This result is summarized in the following proposition.

Proposition 2

Under commercial piracy, the relationship between the rate of piracy and the quality of the pirated product is monotonic i.e. the more reliable the pirated product, the higher is the rate of piracy.

To get a clear intuition of the above result we also do the following analysis. We start with the following results which are used in the intuitive explanation below.

Lemma 3: $\dfrac{d\hat{X}(q)}{dq} > 0$ *; the sign of* $\dfrac{d\hat{Y}(q)}{dq}$ *is indeterminate;* $\dfrac{d\left(\hat{X}(q) - \hat{Y}(q)\right)}{dq} > 0$. (proofs in Appendix 2). [14]

Consider the effect of the increase in the quality of the pirated product on the two marginal consumers under accommodation ($\hat{X}(q)$ and $\hat{Y}(q)$). On one hand, from Lemma 2 (under accommodation) as the quality of the pirated product increases, the original developer decreases the investment to deter the commercial piracy ($\dfrac{\partial x^*}{\partial q} < 0$) and thus the cost of the commercial piracy decreases; this together with the increase in the quality of the pirated product makes the pirated product more attractive. On the other hand, from

[14] $\hat{X}(q)$ and $\hat{Y}(q)$ denote the types of marginal consumers defined in section 2.3 *after* the optimal deterrence effort is plugged into the relevant expressions. Check Appendix 2 to see the exact expressions of $\hat{X}(q)$ and $\hat{Y}(q)$.

Lemma 1 we get the price of the original product decreases ($\frac{dp_o(q)}{dq} < 0$), while the price of the pirated product increases due to the increase in the quality ($\frac{dp_p(q)}{dq} > 0$) and thus makes the pirated product less attractive (call this price difference effect). However, the price difference effect on $\hat{X}(q)$ less than offsets the effect of the increase in the quality of the pirated product as $\frac{d\hat{X}(q)}{dq} > 0$, making the pirated product overall more attractive. As a result, some consumers switch from the original product to the pirated product. This tends to increase the rate of piracy. The effect of the increase of the pirated product's price on $\hat{Y}(q)$ may be less than or more than the effect of the increase in the quality of the pirated product depending on the degree of IPR protection (c) and the diversity of consumers' tastes (θ); since both $\frac{d\hat{Y}(q)}{dq} < 0$ and $\frac{d\hat{Y}(q)}{dq} > 0$ are possible. However, more importantly, it turns out $\hat{X}(q) - \hat{Y}(q)$ always increases in q ($\frac{d\left(\hat{X}(q) - \hat{Y}(q)\right)}{dq} > 0$). We thus get a monotonic relationship between the rate of piracy and the quality of the pirated product under commercial piracy.

4. End-User Piracy

Now we do our analysis for the case of end-user piracy. End-user piracy is quite prevalent, in particular, in the market for digital goods as it is relatively easy to copy a digital product. Here, we assume there is no commercial pirate in the economy, and the consumers (i.e. all potential product users) are the potential pirates. As before, there is one original product developer and a continuum of consumers indexed by X denoting consumer's valuation which is uniformly distributed over the interval $[0, \theta]$ with density $1/\theta$. Consumers have the choice to buy the original product from the product developer or they can pirate themselves. The piracy deterring mechanism of the original product firm remains exactly the same as before, except that now it targets the end user pirates to stop or limit piracy as opposed to commercial pirate by raising their cost of copying. There exists a general level of exogenous IPR protection to reduce piracy in the economy as before. The original firm does not face any direct competition from any commercial pirate in the market; instead, it stands to lose its potential market share because of the end user pirates.[15]

A type-X consumer's utility function is now given as:

$$U = \begin{cases} X - p & \text{if buys original product} \\ qX - (c + x) & \text{if pirates original product} \\ 0 & \text{otherwise,} \end{cases}$$

where x is the level of deterrence for piracy from the original producer and $c > 0$ is the exogenous cost parameter as before measuring the degree of IPR protection targeted to stop/limit end-user piracy.

4.1 Deriving Demand of the Original and Pirated Product

The demand for the original product and for the pirated product, D_o and D_p, can be derived from the distribution of buyers as follows.

The marginal consumer, $\hat{X}$, who is indifferent between buying the original product and pirating is given by $\hat{X} = \frac{p - (c + x)}{1 - q}$. The marginal consumer, $\hat{Y}$, who is indifferent between pirating the product and neither buying original product nor pirating is given by $\hat{Y} = \frac{c + x}{q}$. Thus, the demand for the original firm is $D_o = \frac{1}{\theta}\int_{\hat{X}}^{\theta} dx = \frac{(1-q)\theta - p + (c + x)}{(1-q)\theta}$ and the demand for the pirated product is $D_p = \frac{1}{\theta}\int_{\hat{Y}}^{\hat{X}} dx = \frac{qp - (c + x)}{q(1-q)\theta}$. Here we have implicitly assumed $qp \geq c + x$ so that the demand for the pirate product is nonnegative. When instead $qp \leq c + x$, the developer's demand is $D_o = \frac{\theta - p}{\theta}$.

[15] Here, the assumption of two period time structure is not necessary; everything can be formulated within a single period without loss of generality. There is no strategic game here; it's a monopoly analysis.

Note that since this is a monopoly case, there is no difference between choosing price or quantity by the copyright holder. Without loss of generality, we will allow the copyright holder to choose price.

4.2 Choice of Optimal Price and Level of Deterrence by the Product Developer

When the developer chooses p and x such that $qp \geq c+x$, the firm's profit maximization problem is

$$\max_{p\geq 0, x\geq 0} \pi_o = pD_o - c_o(x) = p\left(\frac{(1-q)\theta - p + (c+x)}{(1-q)\theta}\right) - \frac{1}{2}x^2,$$

$$s.t. \quad qp \geq c+x$$

which is labeled Problem I.

When the developer chooses p and x such that $qp \leq c+x$, the firm's profit maximization problem is

$$\max_{p\geq 0, x\geq 0} \pi_o = pD_o - c_o(x) = p\left(\frac{\theta - p}{\theta}\right) - \frac{1}{2}x^2,$$

$$s.t. \quad qp \leq c+x$$

which is labeled Problem II.

4.2.1 The Optimum

We summarize the optimum in the following proposition after solving Problems I and II (see Appendix 3 for all the details).

Define $\delta(q,\theta) \equiv \dfrac{q(1-q)\theta - 1}{2-q}$.

Proposition 3

(i) *When* $\theta \leq \dfrac{1}{q(1-q)}$ *and* $c < q\theta/2$, *the original developer deters piracy, the optimal price is* $p^* = \dfrac{\theta(1+qc)}{2+q^2\theta}$ *and the optimal level of deterrence is* $x^* = \dfrac{q\theta - 2c}{2+q^2\theta}$.

(ii) *When* $\theta > \dfrac{1}{q(1-q)}$ *and* $c < q\theta/2$,

(a) when $0 \le c \le \delta(q,\theta)$, *the original developer accommodates piracy, the optimal price is* $p^* = \dfrac{(1-q)\theta((1-q)\theta + c)}{2(1-q)\theta - 1}$ *and the optimal level of deterrence is* $x^* = \dfrac{(1-q)\theta + c}{2(1-q)\theta - 1}$.

(b) When $\delta(q,\theta) \le c < q\theta/2$, *the original developer deters piracy, the optimal price is* $p^* = \dfrac{\theta(1+qc)}{2+q^2\theta}$ *and the optimal level of deterrence is* $x^* = \dfrac{q\theta - 2c}{2+q^2\theta}$.

(iii) *When* $c \ge q\theta/2$, *the piracy is blockaded and the original developer's optimal price is the monopoly price* $p^* = \dfrac{\theta}{2}$.

Like in Proposition 1, the condition $\theta \le \dfrac{1}{q(1-q)}$ in Proposition 3(*i*) can be interpreted as when the consumers' tastes are not sufficiently diverse. The intuition for this proposition is qualitatively same as the intuition we provided after proposition 1, hence not repeated here. Note that a necessary condition for entry accommodation to be optimal is $\theta > 4$ since $q(1-q)$ is maximized on the interval (0,1) at *q*=1/2 and the maximum is 1/4. In addition, since $\delta(q,\theta)$ is negative as *q* approaches either to 0 or 1, another necessary condition for entry accommodation to be optimal is that *q* must be intermediate.

4.3 Comparative Statics

4.3.1 Optimal level of deterrence (*x*) and quality of the pirated product (*q*)

Lemma 4: *Under end-user piracy, for entry accommodation we have* $\frac{\partial x^*}{\partial q} > 0$ *and for entry deterrence* : $\frac{\partial x^*}{\partial q} > 0$ if $q < \min\left\{\frac{2c+\sqrt{2\theta+4c^2}}{\theta}, 1\right\}$; $\frac{\partial x^*}{\partial q} < 0$ if $\frac{2c+\sqrt{2\theta+4c^2}}{\theta} < q < 1$ (this implicitly requires $\theta > 2+4c$).

Note that one component of the piracy cost is fixed, namely *c* is constant for a given level IPR protection, so as the quality (*q*) of the pirated good improves the effective piracy cost comes down to the end-user pirates. Thus the copyright holder faces a bigger challenge as *q* increases under end-user piracy.[16] Now under accommodation to recover the loss, the copyright holder needs to be more aggressive and increase the deterrence level as *q* increases. Under deterrence, a similar logic works but there is a further non-monotonicity which can be explained as follows. Note that when $\theta < 2+4c$, $\frac{2c+\sqrt{2\theta+4c^2}}{\theta} > 1$, and the deterrence level increases with *q*. The reason for this aggressive behaviour of the copyright holder is: θ not large enough implies the consumer diversity (heterogeneity) is also not large making the competition tough for the copyright holder as the quality of the pirated good improves, hence to keep piracy out and maintain its market share it increases its level of deterrence. On the other hand, when $\theta > 2+4c$, i.e. when the market is diversified enough similar aggressive behaviour is not needed to stop piracy and hence deterrence level x^* decreases as *q* increases.

To illustrate Proposition 3 and Lemma 4, we provide two numerical examples. In the first example, we fix $\theta = 5$ and $c = 0.1$. Then it is straightforward to obtain the following optimal deterrence effort as a function of *q*:

$$x^* = \begin{cases} 0 & \text{if } 0 < q \le 0.04, \\ (5q-0.2)/(5q^2+2) & \text{if } 0.04 \le q \le 0.36823 \text{ or } 0.65177 \le q < 1, \\ (5.1-5q)/(9-10q) & \text{if } 0.36823 \le q \le 0.65177. \end{cases}$$

[16] Note that things were different under commercial piracy, there as *q* increases, the price of the pirated good from the commercial pirate also increases, hence the cost of using the pirated product increases as well.

In this example, note that unlike the case of commercial piracy (see the illustrative example on commercial piracy in section 3.2.1) when the quality of the pirated product is *intermediate* (medium), the original producer accommodates the commercial pirate. As a result under accommodation,

$$\frac{dx(q)}{dq} = \frac{d}{dq}\left(\frac{5.1-5q}{9-10q}\right) = \frac{6}{(9-10q)^2} > 0 .$$ [17]

This above sign is positive because the pirated good is still far away with respect to the quality from the original one, the competition is less severe, more profit comes to the original producer, which in turn helps the original producer to increase the investment on deterrence. Under complete deterrence,

$$\frac{dx(q)}{dq} = \frac{d}{dq}\left(\frac{5q-0.2}{5q^2+2}\right) = \frac{-25q^2+2q+10}{(5q^2+2)^2}\begin{cases} <0 & \text{for } 0.67372 < q < 1, \\ >0 & \text{for } 0.04 \le q \le 0.36823 \text{ or } 0.65177 \le q < 0.67372. \end{cases}$$

Here similar intuition as above, as long as quality of the pirated good is far away from the original one, the more accruing profit to the original producer helps to increase the investment on piracy deterrence. However, if the pirated product is close to the original product, less profit accrues to the original producer because of severe competition, which leaves little room for further investment on piracy deterrence.

In the second example, we fix $\theta = 5$ and $c = 1$. Then we can get the following optimal deterrence effort as a function of *q*:

$$x^* = \begin{cases} 0 & \text{if } 0 < q \le 0.4, \\ (5q-2)/(5q^2+2) & \text{if } 0.4 \le q < 1. \end{cases}$$

In this example, accommodation is never optimal for the original producer. IPR protection is high enough to deter piracy completely when *q* is small, however strategic entry deterrence is required from the copyright holder for complete deterrence for larger *q*. And under complete deterrence, $\frac{dx(q)}{dq} = \frac{d}{dq}\left(\frac{5q-2}{5q^2+2}\right) = \frac{-25q^2+20q+10}{(5q^2+2)^2} > 0$ for $0.4 \le q < 1$.

[17] This comparative statics sign is negative under commercial piracy. Recall that under commercial piracy, pirate is accommodated only when the quality of the pirated is *high* and consequently *dx(q)/dq < 0*, see section 3.2.1.

4.4 Rate of Piracy and Quality of the Pirated Product

As before, we define the ratio of $D_p/(D_o + D_p)$ to measure the rate of piracy. When $\theta > \dfrac{1}{q(1-q)}$ and $c \le \delta(q,\theta)$ (equivalently, $\dfrac{\theta + c - \sqrt{\theta^2 + c^2 - 6\theta c - 4\theta}}{2\theta} \le q \le \dfrac{\theta + c + \sqrt{\theta^2 + c^2 - 6\theta c - 4\theta}}{2\theta}$), i.e. when the original firm accommodates the pirate, it is straightforward to get $\dfrac{D_p}{D_o + D_p} = \dfrac{q(1-q)\theta - (2-q)c - 1}{2q(1-q)\theta - 2(1-q)c - 1}$.

In all the other cases, entry is either deterred or blockaded; thus, the rate of piracy is zero.

When $\theta > \dfrac{1}{q(1-q)}$ and $c \le \delta(q,\theta)$, simple computation yields

$$\frac{\partial}{\partial q}\left(\frac{D_p}{D_o + D_p}\right) = \frac{(2c+1)\left(c + (1-2q)\theta\right)}{\left(2q(1-q)\theta - 2(1-q)c - 1\right)^2}\begin{cases} > 0 \ \text{ when } \dfrac{\theta + c - \sqrt{\theta^2 + c^2 - 6\theta c - 4\theta}}{2\theta} \le q < \dfrac{\theta + c}{2\theta}, \\ < 0 \ \text{ when } \dfrac{\theta + c}{2\theta} < q \le \dfrac{\theta + c + \sqrt{\theta^2 + c^2 - 6\theta c - 4\theta}}{2\theta}. \end{cases} \quad . \qquad (6)$$

Thus, we have the following proposition.

Proposition 4

Under end user piracy, the relationship between the rate of piracy and the quality of the pirated product is non-monotonic.

To get a clear intuition of the above result again we do a similar analysis as we had done under commercial piracy. We start with the following results which are used in the intuitive explanation below.

Lemma 5: $\dfrac{dp^*}{dq} < 0$, $\dfrac{d\hat{X}^{u}(q)}{dq} < 0$; $\dfrac{d\hat{Y}^{u}(q)}{dq} < 0$ *when q is small and* $\dfrac{d\hat{Y}^{u}(q)}{dq} > 0$ *when q is big*. (proofs are in Appendix 4).[18]

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[18] Explicit expressions of $\hat{X}^{u}(q)$, $\hat{Y}^{u}(q)$ and p^* are written down in Appendix 4.

Like before consider the effect of the increase in the quality of the pirated product on the two marginal consumers under accommodation ($\hat{X}(q)$ and $\hat{Y}(q)$). From Lemma 4 (under accommodation), we know that as the quality of the pirated product increases, the original developer increases the investment to deter the end-user piracy ($\frac{\partial x^*}{\partial q} > 0$) and thus the cost of the end-user piracy increases, and the original developer also decreases the price of its product to attract end-users ($\frac{dp^*}{dq} < 0$), and both of these two effects make the self-pirated product less attractive. These two effects more than offset the effect of the increase in the quality of the pirated product as $\frac{d\hat{X}(q)}{dq} < 0$; making the self-pirated product overall less attractive. Some consumers switch from the self-pirated product to the original product and it tends to decrease the rate of piracy. On the other hand, as the quality of the pirated product increases, the original developer's investment to deter the end-user piracy increases relatively slowly when the quality level is low ($\frac{\partial x^*}{\partial q} = \frac{\theta(2c+1)}{\left(2\theta(1-q)-1\right)^2}$, $\frac{\partial^2 x^*}{\partial q^2} = \frac{4\theta^2(2c+1)}{\left(2\theta(1-q)-1\right)^3} > 0$) and thus the effect of the increase of the cost of the end-user piracy less than offsets the effect of the increase in the quality of the pirated product, as a result, the self-pirated product now becomes attractive to some consumers who neither buy nor pirate the original product before ($\frac{d\hat{Y}(q)}{dq} < 0$). On the contrary, as the quality of the pirated product increases, the original developer's investment to deter the end-user piracy increases relatively quickly when the quality level is high and thus the effect of the increase of the cost of the end-user piracy more than offsets the effect of the increase in the quality of the pirated product, as a result, some consumers switch from the self-pirated product to neither buy nor pirate original product ($\frac{d\hat{Y}(q)}{dq} > 0$). We thus get a non-monotonic relationship between the rate of piracy and the quality of the pirated product under end-user piracy as reflected by (6).

5. Comparison between Commercial Piracy and End-user piracy

In this section, we will do several comparative statics analyses and explore how the nature of piracy as well as competition between the copyright holder and the pirate(s) affect the results. More specifically, here we consider all three scenarios of piracy, namely, (i) commercial piracy under quantity competition, (ii) commercial piracy under price competition (not done here, we will refer to Lu and Poddar (2012)), (iii) end-user piracy under monopoly; to compare the results across these three cases.

5.1 Survival Possibility of Piracy

We study the survival possibility of piracy for any given level of quality of the pirated product q and under any given IPR level of protection c for each scenarios mentioned above. Note that piracy only survives when it is accommodated. Thus, comparing the relevant conditions in Proposition 1 (*ii)(a*), Proposition 3(*ii)(a*), and Proposition 8(ii)(a) (mentioned in Appendix 5) we have the following result.

Proposition 5

Piracy is most likely to survive under commercial piracy and quantity competition; and least likely to survive under end-user piracy.

Proof: See Appendix 6.

This result is a consequence of the fact that the copyright holder faces lower degree of competition from the commercial pirate (in either modes of competition, quantity and price) than from the end-user pirates; thus able to accommodate piracy in a wider range of parametric configuration under commercial piracy. This is a consequence of the fact that under commercial piracy, there is just one pirate who strategically competes and adjusts its price or quantity in response to the original producer's choice of price or quantity with an objective to profitably share the market with the original producer. Whereas in the case of end-user piracy there are numerous pirates (including possibly the very low-valuation consumers), who observe the price of the original producer and make copying decisions to maximize their individual net utility. Naturally, the degree of competition to the copyright holder to preserve its own market is much greater under end-user piracy than commercial piracy.

5.2 Optimal Deterrence Effort by the Copyright Holder

The comparison of the optimal deterrence efforts x^* of the original producer to stop/limit piracy across the three scenarios can be summarized as follows. Since $x^* = 0$ in the blockaded entry case, we exclude it in the following discussion.

Proposition 6

Under deterrence: $x^*(quantity) > x^*(price) = x^*(end-user)$

Under accommodation: $x^*(end-user) > x^*(price) > x^*(quantity)$

Proof: Follows directly from comparing the relevant expressions in Propositions 1, 3 and 8 (in Appendix 5). For the deterrence case, compare the expressions of x^* in Proposition 1*(i)* and *(ii)(b*), with Proposition 3*(i)* and *(ii)(b*), and with Proposition *8(i)* and *(ii)(b)*. For the accommodation case, compare the expressions of x^* in Proposition 1*(ii)(a*), with Proposition 3*(ii)(a*), and with Proposition *8(ii)(a)*.

□

To deter completely under commercial piracy and quantity competition, the original firm has to incur higher effort level as the pirate finds it easiest to survive under this situation compared to other two cases. Also note that in this situation the optimal deterrence effort of the copyright holder is exactly same under price competition and under the case of end user piracy. We believe this is just a coincidence.

On the other hand, for accommodation, since the competition is most relaxed under quantity competition, deterrence effort is the least from the original firm in this situation; whereas it is highest under the end-users piracy case where the competition is severe. The optimal deterrence effort under price competition is in between these two cases as the intensity of the price competition lies in between these two cases as well.

5.3. Relationship between Public and Private anti-piracy Measures

Note that in this analysis the public anti-piracy measure is captured by IPR protection parameter (c) and the private anti-piracy measure is captured by (x) from copyright-holder.

To see the interaction between the two measures, the following comparative statics analysis is useful.

Proposition 7

(i) *For commercial piracy (under either price or quantity competition): Under entry accommodation we have* $\frac{\partial x^*}{\partial c} > 0$*, and under entry deterrence* $\frac{\partial x^*}{\partial c} < 0$.

(ii) *For end-user piracy: Under entry accommodation we have* $\frac{\partial x^*}{\partial c} > 0$*, and under entry deterrence* $\frac{\partial x^*}{\partial c} < 0$.

Proof: The proof is straightforward and omitted here.

Here we show the overall relationship between private and public anti-piracy measures is non-monotonic for both types of piracy. We believe this is an important result from the policy perspective. If we think that stopping or limiting piracy is a joint responsibility of government/public institutions (i.e. IPR laws and enforcements) and private organizations (like an innovative firm or the copyright holder), then the additive piracy deterring cost structure *$(c + x)$* that we have assumed here is rather appropriate and intuitive. Now given this additive structure, from the outset it is only natural to assume that the public effort and the private should be always substitutes in stopping or limiting piracy. However, the above result shows that it may not be the case always. The two efforts can be complements to each other when piracy is accommodated while they are substitutes only when piracy is completely deterred. Thus, to suggest a policy which aims to reduce piracy, the policy makers must take a note of the possibility of non-monotonic relationship between public and private efforts.[19]

[19] We also note the limitation for this finding as c is not chosen endogenously in the model by the government or the IPR authority. An interaction between IPR protection policy i.e. public effort (c) and private effort (x) could be more explicitly captured if c is endogenized. However, in this model, if we just allow the government to choose c with an objective to maximize the overall welfare of the society then the optimal value of c goes

6. Conclusion

Lu and Poddar (2012) analyzed the problem of copyright violations where the copyright holder makes costly investment to deter a commercial pirate in a given regime of intellectual property rights (IPR) protection. We extend the analysis in two directions. Firstly, we change the nature of competition between the copyright holder and the commercial pirate by bringing competition in quantities as opposed to prices. Secondly, instead of assuming a commercial pirate, we do the analysis when the pirates are end-users, thereby changing the nature of piracy.

We find irrespective of the nature of piracy or competition, when the consumers' tastes are sufficiently diverse and IPR protection is weak, it is profitable for the copyright holder to accommodate the pirate(s), while deter the pirate(s) in all other situations. We are also interested to know how the quality of the pirated good impacts the piracy rate as in reality the quality or reliability of the pirated product could vary widely. We find the relationship between the rate of piracy and the quality of the pirated product is monotonically increasing under commercial piracy whereas the relationship is non-monotonic under end-user piracy. We also find the specific relationship between the optimal deterrence effort of the copyright holder and the quality of the pirated under both scenarios of piracy. In general, piracy is more likely to survive under commercial piracy (irrespective of price and quantity competition) than under end-user piracy for any given quality of the pirated product and for a given level of IPR protection. Finally, to find the interaction between two anti-piracy measures i.e. public protection (IPR protection) and private protection (optimal deterrence level from copyright holder), we find non-monotonicity which has policy implications.

Our model has limitations. Here, we did not consider the case, where both kinds of piracy (i.e. commercial and end-user) co-exist in the same market. We do not endogenize the choice of IPR protection policy either. We believe that the model can be extended in these directions. We need to build a unified framework where both types of piracy exist simultaneously and where the government or IPR protection authorities play a more pro-active role in controlling piracy by monitoring and penalizing the pirate(s) and thus IPR

to zero, a conclusion which is neither interesting nor realistic. We need to have a revised government objective (not just simple welfare maximization), to get an optimal c which is positive and meaningful.

protection policy is endogenized. On another dimension, it would be also interesting to see as a country grows from a lower income to a higher income nation, how the composition of the piracy (between commercial and end-user) in the society and the piracy rates change endogenously as a response to IPR and private protection policies. We want to pursue all these issues in our future research.

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Appendices

Appendix 1

$$p_o(q,x)=\frac{1}{4-q}\left(c+x+(2-q)\theta\right),\quad p_p(q,x)=\frac{(2-q)(c+x)+q\theta}{4-q}.$$

Observation: $\frac{\partial p_o(q,x)}{\partial q}<0$ and $\frac{\partial p_p(q,x)}{\partial q}>0$.

Proof:

Since $2(c+x)\le q\theta$, $\frac{\partial}{\partial q}p_o(q,x)=\frac{c+x-2\theta}{(4-q)^2}<0$, $\frac{\partial}{\partial q}p_p(q,x)=-2\frac{c+x-2\theta}{(4-q)^2}>0$.

□

$$p_o(q)=\frac{\theta(4-q)(c+(2-q)\theta)}{(4-q)^2\theta-2},\quad p_p(q)=\theta\frac{c(2-q)(4-q)+q(4-q)\theta+2(1-q)}{(4-q)^2\theta-2}.$$

Lemma 1: $\frac{dp_o(q)}{dq}<0$ and $\frac{dp_p(q)}{dq}>0$.

Proof:

$$\frac{dp_o(q)}{dq}=\frac{c\left((4-q)^2\theta-2\right)-2\theta\left((4-q)^2\theta+2q-6\right)}{\left((4-q)^2\theta-2\right)^2}$$

$$\le\frac{\frac{q(4-q)\theta-2}{2(4-q)}\left((4-q)^2\theta-2\right)-2\theta\left((4-q)^2\theta+2q-6\right)}{\left((4-q)^2\theta-2\right)^2}=-\frac{1}{2(4-q)}<0.$$

$\frac{dp_p(q)}{dq}=-\frac{2\theta}{\left((4-q)^2\theta-2\right)^2}\left(c\left((4-q)^2\theta+2q-6\right)-2\theta^2(4-q)^2+\theta\left(12-q^2\right)-2\right)$. Since

$\theta>\frac{2}{q(4-q)}>\frac{12-q^2}{4(4-q)^2}$,

$$-2\theta^2(4-q)^2+\theta\left(12-q^2\right)-2<-2\left(\frac{2}{q(4-q)}\right)^2(4-q)^2+\frac{2}{q(4-q)}\left(12-q^2\right)-2=-8\frac{(2-q)^2}{q^2(4-q)}<0.$$

If $(4-q)^2\theta+2q-6<0$, then $\frac{dp_p(q)}{dq}>0$; if $(4-q)^2\theta+2q-6>0$, then

$$c\left(\left(4-q\right)^2\theta+2q-6\right)-2\theta^2\left(4-q\right)^2+\theta\left(12-q^2\right)-2$$

$$\leq\frac{q\left(4-q\right)\theta-2}{2\left(4-q\right)}\left(\left(4-q\right)^2\theta+2q-6\right)-2\theta^2\left(4-q\right)^2+\theta\left(12-q^2\right)-2=-\frac{\left(\left(4-q\right)^2\theta-2\right)^2}{2\left(4-q\right)}<0,$$

and thus $\frac{dp_p\left(q\right)}{dq}>0$. □

Appendix 2

Lemma 3: $\frac{d\tilde{K}(q)}{dq}>0$*; the sign of* $\frac{d\tilde{Y}(q)}{dq}$ *is indeterminate;* $\frac{d\left(\tilde{K}(q)-\tilde{Y}(q)\right)}{dq}>0$.

We write the expressions of $\tilde{K}(q)$ and $\tilde{Y}(q)$ after the optimal deterrence effort is plugged into the relevant expressions first:

$$\tilde{K}(q)=\frac{\theta}{\left(4-q\right)^2\theta-2}\left(-4c+8\theta-2-2q\theta+cq\right),$$

$$\tilde{Y}(q)=\frac{\theta}{q\left(\left(4-q\right)^2\theta-2\right)}\left(8c-2q+4q\theta+cq^2-q^2\theta-6cq+2\right),\text{ and thus}$$

$$\tilde{K}(q)-\tilde{Y}(q)=\frac{\theta}{q\left(\left(4-q\right)^2\theta-2\right)}\left(-8c+4q\theta-q^2\theta+2cq-2\right).$$

Proof:

(1) $\frac{d\tilde{K}(q)}{dq}=\frac{-\theta}{\left(\left(4-q\right)^2\theta-2\right)^2}\left(c\left(\left(4-q\right)^2\theta+2\right)-2\left(4-q\right)^2\theta^2+4\left(3-q\right)\theta\right)>0$ since

$c\leq\frac{q\left(4-q\right)\theta-2}{2\left(4-q\right)}$ and thus

$$c\left(\left(4-q\right)^2\theta+2\right)-2\left(4-q\right)^2\theta^2+4\left(3-q\right)\theta$$

$$\leq\frac{q\left(4-q\right)\theta-2}{2\left(4-q\right)}\left(\left(4-q\right)^2\theta+2\right)-2\left(4-q\right)^2\theta^2+4\left(3-q\right)\theta=-\frac{\left(\left(4-q\right)^2\theta-2\right)^2}{2\left(4-q\right)}<0.$$

(2) $\frac{d\hat{Y}(q)}{dq}=\frac{-\theta}{q^2\left(\left(4-q\right)^2\theta-2\right)^2}\left(f\left(\theta\right)c+g\left(\theta\right)\right)$, where

$f\left(\theta\right)\equiv\left(q^2-4q+8\right)\left(4-q\right)^2\theta+2q^2-16$ and

$g\left(\theta\right)\equiv-q^2\left(4-q\right)^2\theta^2-4\theta\left(8q-5q^2+q^3-8\right)-4$. Consider the sign of $f\left(\theta\right)$. Clearly, it is increasing in θ. Since under accommodation $\theta>\frac{2}{q\left(4-q\right)}$, $f\left(\theta\right)>f\left(\frac{2}{q\left(4-q\right)}\right)=\frac{16\left(2-q\right)^2}{q}>0$.

Now consider the sign of $g\left(\theta\right)$. Note that $g\left(\frac{2}{q\left(4-q\right)}\right)=\frac{8\left(2-q\right)^3}{q\left(4-q\right)}>0$ and that $g\left(\theta\right)<0$ when θ is sufficiently big. Since under accommodation $\theta>\frac{2}{q\left(4-q\right)}$, $g\left(\theta\right)$ can be positive or negative, depending on the value of θ. Finally, we note that

$f\left(\theta\right)c+g\left(\theta\right)<f\left(\theta\right)\frac{q\left(4-q\right)\theta-2}{2\left(4-q\right)}+g\left(\theta\right)=\frac{q\left(2-q\right)\left(\left(4-q\right)^2\theta-2\right)^2}{2\left(4-q\right)}$ since under accommodation $c<\frac{q\left(4-q\right)\theta-2}{2\left(4-q\right)}$. So we conclude that the sign of $\frac{d\hat{Y}(q)}{dq}$ is indeterminate, i.e. it can be positive or negative. For example, when θ is sufficiently big and *c* is sufficiently small, $\frac{d\hat{Y}(q)}{dq}<0$; when *c* is sufficiently close to $\frac{q\left(4-q\right)\theta-2}{2\left(4-q\right)}$, $\frac{d\hat{Y}(q)}{dq}>0$.

(3)

$$\frac{d}{dq}\left(\hat{X}(q)-\hat{Y}(q)\right)=\frac{\theta}{q^2\left(\left(4-q\right)^2\theta-2\right)^2}\left(q^2\left(4-q\right)^2\theta^2+8\left(2-q\right)^2\theta-16c-4+4\left(2-q\right)\left(4-q\right)^2\theta c\right).$$

Since $\theta>\frac{2}{\left(4-q\right)q}$ and $c<\frac{q\left(4-q\right)\theta-2}{2\left(4-q\right)}$ under accommodation, we have

$$q^2\left(4-q\right)^2\theta^2+8\left(2-q\right)^2\theta-16c-4+4\left(2-q\right)\left(4-q\right)^2\theta c$$
$$>4+8\left(2-q\right)^2\theta-16c-4+4\left(2-q\right)\left(4-q\right)^2\theta c=8\left(2-q\right)^2\theta+\left(4\left(2-q\right)\left(4-q\right)^2\theta-16\right)c$$
$$>8\left(2-q\right)^2\theta+\left(4\left(2-q\right)\left(4-q\right)^2\frac{2}{\left(4-q\right)q}-16\right)c=8\left(2-q\right)^2\theta+\frac{8}{q}\left(8-8q+q^2\right)c>0.$$

Therefore, $\frac{d}{dq}\left(X^{u}(q)-Y^{u}(q)\right)>0$.

□

Appendix 3

Problem I

Define Lagrangian $L_1\left(p,x,\lambda\right)=p\left(\frac{(1-q)\theta-p+\left(c+x\right)}{\left(1-q\right)\theta}\right)-\frac{1}{2}x^2+\lambda\left(qp-c-x\right)$.

The sufficient and necessary conditions for the optimum are the following:

$$\frac{\partial L_1\left(p,x,\lambda\right)}{\partial p}=\frac{(1-q)\theta-2p+\left(c+x\right)}{\left(1-q\right)\theta}+\lambda q=0, \tag{A1}$$

$$\frac{\partial L_1\left(p,x,\lambda\right)}{\partial x}=\frac{p}{\left(1-q\right)\theta}-x-\lambda=0, \tag{A2}$$

$$\lambda\left(qp-c-x\right)=0,\ \lambda\geq 0,\ qp\geq c+x. \tag{A3}$$

If $\lambda=0$, then we can solve for *p* and *x* from (A1) and (A2) after plugging $\lambda=0$ into these equations and get $p=\frac{(1-q)\theta\left((1-q)\theta+c\right)}{2\left(1-q\right)\theta-1}$ and $x=\frac{(1-q)\theta+c}{2\left(1-q\right)\theta-1}$. We also need to check whether $qp\geq c+x$ is satisfied and we find that this condition is satisfied when $c\leq\frac{q\left(1-q\right)\theta-1}{2-q}$. In this case, the developer's profit is $\pi_o^{\ A}=\frac{\left((1-q)\theta+c\right)^2}{2\left(2\left(1-q\right)\theta-1\right)}$, where the superscript A indicates this is an accommodation case.

If instead $qp=c+x$, then we can solve for *p* and *x* from (A1), (A2) and $qp=c+x$, and get $p=\frac{\theta\left(1+qc\right)}{2+q^2\theta}$, and $x=\frac{q\theta-2c}{2+q^2\theta}$. Note that $x\geq 0$ when $c\leq\frac{q\theta}{2}$. We also need to check whether $\lambda\geq 0$ is satisfied and we find that this condition is satisfied when

$c \geq \frac{q(1-q)\theta - 1}{2-q}$. In this case, the developer's profit is $\pi_o{}^D = \frac{\theta + 2c(q\theta - c)}{2(2+q^2\theta)}$, where the superscript D indicates this is a deterrence case.

Problem II

Define Lagrangian $L_2(p,x,\kappa) = p\left(\frac{\theta - p}{\theta}\right) - \frac{1}{2}x^2 - \kappa(qp - c - x)$. The sufficient and necessary conditions for the optimum are the following:

$$\frac{\partial L_2(p,x,\kappa)}{\partial p} = \frac{\theta - 2p}{\theta} - \kappa q = 0, \tag{A4}$$

$$\frac{\partial L_2(p,x,\kappa)}{\partial x} = -x + \kappa = 0, \tag{A5}$$

$$\kappa(qp - c - x) = 0,\ \kappa \geq 0,\ qp \leq c + x. \tag{A6}$$

If $\kappa = 0$, then we can solve for *p* and *x* from (A4) and (A5) after plugging $\kappa = 0$ into these equations and get $p = \frac{\theta}{2}$ and $x = 0$. We also need to check whether $qp \leq c + x$ is satisfied and we find that this condition is satisfied when $c \geq \frac{q\theta}{2}$. This is clearly the blockade case since the condition $qp \leq c + x$ is satisfied when the original developer chooses the monopoly price, $p = \frac{\theta}{2}$, and zero deterrence level, *x*=0. In this case, the developer's profit is $\pi_o{}^B = \frac{\theta}{4}$, where the superscript B indicates this is a blockade case.

If instead $qp = c + x$, then we can solve for *p* and *x* from (A4), (A5) and $qp = c + x$, and get $p = \frac{\theta(1+qc)}{2+q^2\theta}$, and $x = \frac{q\theta - 2c}{2+q^2\theta}$. We also need to check whether $\kappa \geq 0$ is satisfied and we find that this condition is satisfied when $c \leq \frac{q\theta}{2}$. This is clearly the deterrence case and the developer's profit is $\pi_o{}^D = \frac{\theta + 2c(q\theta - c)}{2(2+q^2\theta)}$.

□

Appendix 4

Lemma 5: $\frac{dp^*}{dq}<0$, $\frac{d\hat{k}(q)}{dq}<0$*;* $\frac{d\hat{Y}(q)}{dq}<0$ *when q is small and* $\frac{d\hat{Y}(q)}{dq}>0$ *when q is big.*

We write the expression of p^* and the expressions of $\hat{k}(q)$ and $\hat{Y}(q)$ after the optimal deterrence effort is plugged into the relevant expressions first:

$$p^*=\frac{(1-q)\theta\big((1-q)\theta+c\big)}{2(1-q)\theta-1},\quad \hat{k}(q)=\frac{\theta}{2(1-q)\theta-1}\big((1-q)\theta-1-c\big), \text{ and}$$

$$\hat{Y}(q)=\frac{\theta(1-q)(2c+1)}{q\big(2(1-q)\theta-1\big)}.$$

Proof: (1)

$$\frac{dp^*}{dq}=\theta\frac{c-2(1-q)\theta\big((1-q)\theta-1\big)}{\big(2(1-q)\theta-1\big)^2}\le\frac{\theta}{\big(2(1-q)\theta-1\big)^2}\left(\frac{q(1-q)\theta-1}{2-q}-2(1-q)\theta\big((1-q)\theta-1\big)\right)$$

$$=-\frac{\theta\big((2-q)(1-q)\theta-1\big)}{\big(2(1-q)\theta-1\big)(2-q)}<0.$$

The first inequality holds since under accommodation $c\le\frac{q(1-q)\theta-1}{2-q}$.

(2) $\frac{d\hat{k}(q)}{dq}=-\theta^2\frac{2c+1}{\big(2(1-q)\theta-1\big)^2}<0$.

(3)

$$\frac{d\hat{Y}(q)}{dq}=\frac{\theta(2c+1)\big(1-2\theta(1-q)^2\big)}{\big(2(1-q)\theta-1\big)^2}\begin{cases}<0 & \text{for } \frac{\theta+c-\sqrt{\theta^2+c^2-6\theta c-4\theta}}{2\theta}\le q<1-\sqrt{\frac{1}{2\theta}},\\ >0 & \text{for } 1-\sqrt{\frac{1}{2\theta}}<q\le\frac{\theta+c+\sqrt{\theta^2+c^2-6\theta c-4\theta}}{2\theta}.\end{cases}$$

□

Appendix 5

Price Competition

Following Lu and Poddar (2012), here we just summarize the entry accommodation equilibrium and entry deterrence equilibrium.

Define:

$$\eta(q,\theta)\equiv\frac{q(1-q)(16-12q+q^2)\theta+6q-8-q\sqrt{(1-q)(2+q^2\theta)\left((4-q)^2(1-q)\theta-2\right)}}{2(2-q)(8-8q+q^2)}.$$

Proposition 8

(i) *When* $q(4-q)(1-q)\theta\le 2$ *and* $c<q\theta/2$*, the original producer's optimal level of deterrence is* $x^*=(q\theta-2c)/(q^2\theta+2)$*. In this case, it deters the pirate and the pirate has no demand.*

(ii) *When* $q(4-q)(1-q)\theta>2$ *and* $c<q\theta/2$*,*

(a) When $c\le\eta(q,\theta)$*, the original producer's optimal level of deterrence is* $x^*=2(c+2(1-q)\theta)/\left((4-q)^2(1-q)\theta-2\right)$*. In this case, it accommodates the pirate and shares the market with the pirate.*

(b) When $\eta(q,\theta)\le c<q\theta/2$*, the original producer's optimal level of deterrence is* $x^*=(q\theta-2c)/(q^2\theta+2)$*. In this case, it deters the pirate and the pirate has no demand.*

(iii) *When* $c\ge q\theta/2$*, there is no need to deter the pirate strategically. Piracy is blockaded anyway due to exogenous high level of IPR protection.*

Appendix 6

From Proposition 1, under commercial piracy and quantity competition, accommodation occurs when $c\le\phi(q,\theta)\equiv\frac{q(4-q)\theta-2}{2(4-q)}$ and $\theta>\frac{2}{q(4-q)}$; from Proposition 3, under end-user piracy, accommodation occurs when $c\le\delta(q,\theta)\equiv\frac{q(1-q)\theta-1}{2-q}$ and $\theta>\frac{1}{q(1-q)}$, and from Proposition 8 (see appendix 5), under commercial piracy and price competition, accommodation occurs when $c\le\eta(q,\theta)$ and $q(4-q)(1-q)\theta>2$. Now, given $\delta(q,\theta)<\eta(q,\theta)<\phi(q,\theta)$ and $(4-q)^2\theta>q(4-q)(1-q)\theta>2q(1-q)\theta$, the result follows.