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Strategic Choice of Network Externality

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Strategic Choice of Network Externality*

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Abstract

In many product markets, impact of network externality plays an important role to affect the overall quality of a product. However, the degree or the strength of network externality is assumed as a parameter in most of the literature. We propose a model of vertical product differentiation with two competing firms where the strength of network externality is endogenized as a strategic choice of the high quality firm. We show how the equilibrium market structure and market coverage depend on the cost of choosing the network strength and on the relative quality difference of the competing products. We also show that the relationship between the optimal level of network externalities and the relative quality differences of the products can be monotonic or non-monotonic.

Keywords: Vertical product differentiation, Network externality, Market structure, Market coverage, Investment cost

JEL Classifications: D23, D43, L13, L86

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1. Introduction

In the modern environment of digital products market with strong features of network externalities, a leading firm in this industry is not only just interested to increase the size of its users-base i.e. the network size of its products, it is also interested to enhance the *strength* of the network effect by making appropriate investments, which consequently increases the overall value of the products. Adding new apps, features and supporting devices to its core product over time enhances the usage and overall quality of the product. This is an alternative way to enhance the effective quality of an existing product without investing to improve the intrinsic quality of the product which may not also be feasible in many circumstances. In this paper, we study this particular aspect through a model of vertical product differentiation with two competing firms. There is one high quality and one low quality firm. The high quality firm strategically invests to improve the strength or the degree of network externality to increase the overall value of its core product for its users. The low quality firm also benefits (and free rides) from the effective increase of the network strength for its product. However, due to intrinsic lower quality of its product, the absorption capacity of the network effects is limited. A consequence of this is, the high quality product generates more value (apart from its inherent high quality) to its users compared to the low quality product. Thus, it also gives adequate incentive for the high quality firm to invest in increasing the strength of network externality. In this set up we study the strategic interaction between the two firms in the product market and see its impact on the equilibrium market structure and market coverage.[1]

To capture the situation, we consider a two-stage game where in the first stage, the high quality firm makes costly investment to improve or enhance the impact of the network externality. In the second stage it competes in price with the low quality producer. We solve the two-stage game and find out the impact of investment cost on the network strength chosen by the high quality firm for any given level of relative quality difference between the products. To get the main idea in a simplified framework we first consider two levels of investment cost on the network, namely, low and high. We first find out what would be the optimal levels or strengths of the network externalities for the high quality firm under low and high costs environment. Then

[1] Most digital products fall in this category where bringing new accessories, supporting devices and suitable applications (apps) to the core product that connect different users enhance the strength of the network effect and hence improve the effective quality of the core product.

for each optimal level of network externality under a given cost environment, we find out what would be the ensuing equilibrium market structure (i.e. whether market outcome is monopoly or duopoly) and whether the market will be partially or fully covered in the equilibrium. Thus, to this end we endogenize these two important aspects of the market.

We find when the cost of investment on the network is low and the quality difference between the two products is high, the market is always monopolized and fully covered by the high quality firm, while it remains duopoly when the quality difference is low. On the other hand, when the cost of investment on the network is high, the market always remains duopoly, however, it may be partially or fully covered. From the comparative statics analysis, we find that under low cost environment, the optimal level of network externality is always increasing in quality difference of the products, and the relationship is monotonic. Under high cost environment even if the optimal level of network externality is always increasing in quality difference piecewise, however, the overall relationship between the two is non-monotonic.

We then generalize the analysis to find the optimal degrees of network externalities chosen by the high quality firm under all possible cost scenrios. We find the equilibrium market structures and market coverage under low, low-medium, medium and high investment costs on the network that essentially captures all possible cost scenarios needed for a complete equilibrium characterization of the problem.[2] We find that the market can be monopolized and fully covered by the high quality firm when the investment cost on the network is low or low-medium and the quality difference between the two products is high, otherwise the market mostly remains duopoly, which may be partially or fully covered. Secondly, if the investment cost on the network is very low or very high, the optimal degree of network externality is always increasing in quality difference of the products, but for any other levels of investment cost the relationship is non-monotonic although the relationship is non-decreasing piecewise.

The impact of network externality or network effects on various economic situations is studied in the literature in several contexts. There is a sizeable research discussing the effects of network externality in economic contexts which originated from the studies by Rohlfs (1974), Katz and Shapiro (1985), followed by Chou and Shy (1990), Church and Gandal (1993) among many others. Grilo et. al (2001) modelled social pressures such as conformity or vanity in terms

[2] Note that the low and high cost scenarios in the simplified framework is a sub-case of the general framework that correspond to low and medium cost scenarios respectively.

of consumption externalities in a model of spatial duopoly and characterized various possible equilibrium outcomes and market structures. Lambertini and Orsini (2005) considered a model of vertical product differentiation with the feature of positive network externality and focused on the existence of quality-price equilibrium. In another area, where the impact of network externality is getting a renewed interest is copyright violations or piracy in the digital products market (see the work done by Conner and Rumelt 1991, Takeyama 1994, Shy and Thisse 1999, Banerjee 2003, 2013 among others). However, the research dealing with the impact of network externality on consumer and producer behaviour in all the above studies assumes the degree or strength of the network effect as an exogenous parameter in the analysis. But it is apparent that from a firm's point of view, improving the network strength of its product has to be a part of strategic decision making process apart from the pricing decisions.

The rest of the paper is organized as follows. In section 2, we present the basic model. Sections 3 and 4 analyze the price competition between the two firms when the market is partially and fully covered respectively, assuming the degree of demand network externality as a parameter. In section 5, we endogenize the degree of network externality under the low and high network cost environment. The high quality firm strategically chooses the optimal degree of network externality; and the market structure and market coverage are endogenously determined in the equilibrium. In section 6, we generalize the analysis under all possible costs environments and summarize the main findings. Section 7 concludes.

2. The Model

A two-stage game is considered in a vertically differentiated product market with two firms. The high quality product is denoted by H and the low quality product is denoted by L. There is a continuum of consumers distributed uniformly over a unit interval with heterogeneous preferences towards product quality. The products also exhibit the feature of positive network externality.[3] However, as explained before, the impact of the network externality is *asymmetric* between the users of the high quality product and low quality product. The users of the high quality product enjoy a higher level of network externality compared to the users of the low

[3] We rule out the case of any negative network externality in this model.

quality product as the absorption capacity of the network effect of the high quality product is more than that of the low quality product.[4]

In terms of utility, the consumer who buys the high quality product, first of all, gets all the intrinsic benefit from the product due to its high quality; secondly she also enjoys the full extent of the network externality generated by those users who also buy the high quality product, plus the (limited) network externality generated by the low quality product users. The buyers of the low quality product can enjoy all the value of the product (intrinsic as well as network) subject to limitation that the lower quality can permit. We normalize the quality of the high quality firm's product to one. The low quality product is indexed by q, $q \in (0,1)$ where q captures the quality depreciation.

Formally, the utility of a typical consumer X ($X \in [0,1]$) is given as follows: [5]

$$U = \begin{cases} X + \gamma D_H + q\gamma D_L - p_H & \text{if buys high quality product,} \\ q\left(X + \gamma D_H + \gamma D_L\right) - p_L & \text{if buys low quality product,} \\ 0 & \text{if buys none,} \end{cases}$$

where D_H, p_H and D_L, p_L are the demand and prices for the high quality and low quality products respectively. $\gamma > 0$ is the coefficient which measures the *level or strength* of network externalities. For example, higher γ implies stronger effect of network externality, whereas when γ is close to zero, it implies almost no effect of network externality. In our model only the high quality firm can influence γ. The interpretation is choosing a higher level of γ is same as choosing more useful features, apps or supporting devices for the core product to enhance the strength of the network externality by improving connectivity among the users.

In stage 1, the high quality firm chooses how much to invest to improve the strength of network externality, i.e. chooses a level of γ. In stage 2, the high quality firm and the low quality firm compete in prices in the product market for a given level of γ. The high quality firm can choose the level of network externality γ by incurring a cost of $c(\gamma) = \frac{1}{2}k\gamma^2$, where

[4] High quality product has all latest features functional to absorb all the network effect, while the low quality product has limited functionality and absorption capacity of the network effect.

[5] The utility representation is borrowed from the standard model of vertical product differentiation in the literature (see Shaked and Sutton, 1982; Tirole, 1988, see also Banerjee 2003). The parameter *q* can be interpreted as a quality index for the low quality product.

$k > 0$ measures how costly it is to invest in enhancing the level of network externality. For simplicity, we assume the costs of production for the firms are zero. The size of the market is normalized to 1. We look for the subgame perfect Nash equilibrium of this game and work backward.

Consider the price competition stage. We will first deal with cases when the market is partially covered and when it is fully covered separately. The market coverage aspect gets endogenized later when γ is chosen.

3. Partial Market Coverage

The marginal consumer X_m, who is indifferent between buying the high quality and low quality product, is given by

$$X_m + \gamma D_H + q\gamma D_L - p_H = q\left(X_m + \gamma D_H + \gamma D_L\right) - p_L .$$

This gives us $X_m = \dfrac{p_H - p_L}{1-q} - \gamma D_H$. The marginal consumer Y_m, who is indifferent between buying the low quality product and buying none, is given by

$$q\left(Y_m + \gamma D_H + \gamma D_L\right) - p_L = 0 .$$

We thus have $Y_m = \dfrac{p_L}{q} - \gamma\left(D_H + D_L\right)$.

The demand for the high quality product is given by $D_H = 1 - X_m$; we then obtain

$$D_H = \frac{1}{1-\gamma}\left[1 - \frac{p_H - p_L}{1-q}\right]. \qquad (1)$$

The demand for the low quality product is given by:

$$D_L = X_m - Y_m = \frac{p_H q - p_L}{q(1-q)(1-\gamma)} .^{6} \qquad (2)$$

The high and low quality firms compete by choosing prices strategically. The Nash equilibrium prices and demands are

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[6] To be precise, this demand system is correct only when $p_H q \geq p_L$. However, it can be shown that in equilibrium we do have $p_H q > p_L$. For the sake of simplicity, here we omit the demand system when $p_H q < p_L$.

$$p_H^* = \frac{2(1-q)}{4-q}, \ p_L^* = \frac{q(1-q)}{4-q}, \ D_H^* = \frac{2}{(1-\gamma)(4-q)}, \ D_L^* = \frac{1}{(1-\gamma)(4-q)}.$$

The profits of the high and low quality firms are respectively

$$\pi_H^* = \frac{4(1-q)}{(1-\gamma)(4-q)^2}, \ \pi_L^* = \frac{q(1-q)}{(1-\gamma)(4-q)^2}.$$

Under this case, the market structure is always duopoly.

Next we deal with the case of full market coverage.

4. Full Market Coverage

Note that we have normalized the size of the market to be 1. There will be an upper bound of the level of network effect γ for which $D_H^* + D_L^* = 1$. Denote that upper bound by $\hat{\gamma}$. From the previous analysis, we find $\hat{\gamma} = \frac{1-q}{4-q}$. So when $\gamma \geq \hat{\gamma}$, the market is always fully covered.

Now, when the market is fully covered, we have the following analysis. First, in the case of full market coverage, we can distinguish two cases: (i) both the firms are active (i.e. duopoly); and (ii) only the high quality firm serves the whole market (i.e. monopoly) Case (ii) arises when γ is above a threshold value denoted by $\tilde{\gamma}$ (which will be defined later), and case (i) realizes when $\hat{\gamma} \leq \gamma < \tilde{\gamma}$. When case (ii) arises the market actually gets back to monopoly from duopoly as we will see that the low quality firm cannot sell its product even if it sets its price at marginal cost. Thus, market monopolization happens endogenously here as the strength of the network effect γ crosses a certain threshold.

Now we will analyze case (i) in Section 4.1 and case (ii) in Section 4.2 under the full market coverage in detail.

4.1 $\hat{\gamma} \leq \gamma < \tilde{\gamma}$ (Moderate level of network externality)

Here, the demand for the high quality firm remains same as in the previous case, i.e. equation (1), while the demand for the low quality firm is

$$D_L = 1 - D_H = \frac{1}{1-\gamma}\left(\frac{p_H - p_L}{1-q} - \gamma\right). \tag{3}$$

Note that in this case, full coverage of the market implies the consumer with *X=0* also obtains nonnegative surplus, which can be expressed as $q\gamma - p_L \geq 0$. It turns out we need to distinguish two subcases (a) and (b). In subcase (a) the consumer with *X=0* has no surplus while the consumer gets positive surplus in subcase (b).

The Nash equilibrium prices, demands and each firm's profit are given below. The detailed analysis is presented in Appendix A.

Subcase (a) When $\frac{1-q}{4-q} = \hat{\gamma} \leq \gamma \leq \hat{\hat{\gamma}} = \frac{1-q}{2+q}$

$$p_H^* = \frac{1-q+q\gamma}{2}, \quad p_L^* = q\gamma, \; D_H^* = \frac{(1-q)+q\gamma}{2(1-q)(1-\gamma)}, D_L^* = \frac{(1-q)-(2-q)\gamma}{2(1-q)(1-\gamma)}, \; \pi_H^* = \frac{\left((1-q)+q\gamma\right)^2}{4(1-q)(1-\gamma)},$$

$$\pi_L^* = q\gamma \frac{(1-q)-(2-q)\gamma}{2(1-q)(1-\gamma)};$$

Subcase (b) When $\frac{1-q}{2+q} = \hat{\hat{\gamma}} \leq \gamma \leq \tilde{\gamma} = \frac{1}{2}$

$$p_H^* = \frac{1-q}{3}(2-\gamma), \quad p_L^* = \frac{1-q}{3}(1-2\gamma), \; D_H^* = \frac{2-\gamma}{3(1-\gamma)}, \; D_L^* = \frac{1-2\gamma}{3(1-\gamma)}, \; \pi_H^* = \frac{(1-q)(2-\gamma)^2}{9(1-\gamma)},$$

$$\pi_L^* = \frac{(1-q)(1-2\gamma)^2}{9(1-\gamma)}.$$

4.2 $\gamma \geq \tilde{\gamma}$ (High level of network externality)

When $\gamma = \tilde{\gamma} = \frac{1}{2}$, from the expression of the price and demand of the low quality firm (in the previous section 4.1, subcase (b)), we find that they go to zero (hence the profit as well). This means when γ reaches that threshold or beyond, the low quality firm is unable to compete profitably, market becomes monopoly and all consumers buy the high quality product even if the low quality product is free (strictly speaking, sold at marginal cost). To ensure the consumer with the lowest *X*(*=0*) buys the high quality product, the following condition has to be satisfied: $\gamma D_H - p_H \geq q(\gamma D_H + \gamma D_L) - p_L \Leftrightarrow \gamma - p_H \geq q\gamma$ (since $D_H = 1, D_L = 0, p_L = 0$). It is not hard to see that the high quality firm charges a price $(1-q)\gamma$ to maximize its profit and its total profit is also $(1-q)\gamma$ (recall that the size of the market is normalized to 1). This is true irrespective of the quality of the low quality product.

5. Choice of Optimal Level of Network Externality under Low and High Cost Environment

The main purpose of this section is to show what type of market structure and the market coverage come out endogenously in equilibrium when the level of network externality is chosen by the high quality firm strategically. As we are focusing on two cost environments (low and high) for the high quality firm in choosing network strength, it suffices to choose a lower value of *k* i.e. *k*=1 to represent low cost environment and a higher value *k*=2 for high cost environment. We will see later that these two parameters values of *k* can capture all possible market structures and market coverage that may arise in equilibrium. So it is sufficient to focus on these two cases without any loss of generality of the problem.

Now according to the analysis in Sections 3 and 4, the high quality firm's profit in the second stage can be summarized as

$$\pi_H^*(\gamma)=\begin{cases}\dfrac{4(1-q)}{(1-\gamma)(4-q)^2} & \text{if } 0\le\gamma\le\dfrac{1-q}{4-q},\\[2ex] \dfrac{\left((1-q)+q\gamma\right)^2}{4(1-q)(1-\gamma)} & \text{if } \dfrac{1-q}{4-q}\le\gamma\le\dfrac{1-q}{2+q},\\[2ex] \dfrac{(1-q)(2-\gamma)^2}{9(1-\gamma)} & \text{if } \dfrac{1-q}{2+q}\le\gamma\le\dfrac{1}{2},\\[2ex] (1-q)\gamma & \text{if } \gamma\ge\dfrac{1}{2}.\end{cases}$$

In Stage 1, the high quality firm chooses γ to maximize its net profit $\pi^N(\gamma)=\pi_H^*(\gamma)-\frac{1}{2}k\gamma^2$. Note that $\pi^N(\gamma)$ is continuous in γ.

Denote $f(\gamma)=\dfrac{4(1-q)}{(1-\gamma)(4-q)^2}-\dfrac{1}{2}k\gamma^2$ for $\gamma\le\dfrac{1-q}{4-q}$, $g(\gamma)=\dfrac{\left((1-q)+q\gamma\right)^2}{4(1-q)(1-\gamma)}-\dfrac{1}{2}k\gamma^2$ for $\dfrac{1-q}{4-q}\le\gamma\le\dfrac{1-q}{2+q}$, $h(\gamma)=\dfrac{(1-q)(2-\gamma)^2}{9(1-\gamma)}-\dfrac{1}{2}k\gamma^2$ for $\dfrac{1-q}{2+q}\le\gamma\le\dfrac{1}{2}$, and $j(\gamma)=(1-q)\gamma-\dfrac{1}{2}k\gamma^2$

for $\gamma \geq \frac{1}{2}$. To find the optimal level of network externality γ^*, we need to find the maximum of $f(\gamma)$, $g(\gamma)$, $h(\gamma)$ and $j(\gamma)$ in the relevant ranges of γ and then compare these maximums.

5.1 Low Cost Environment: *k*=1

Consider $f(\gamma)$ first. We can easily show that $f'(\gamma)=\gamma\left(\frac{4(1-q)}{\gamma(1-\gamma)^2(4-q)^2}-1\right)>0$ in the relevant range $0 \leq \gamma \leq \frac{1-q}{4-q}$. Therefore, $f(\gamma)$ is maximized at $\gamma = \frac{1-q}{4-q}$.

Next we examine $g(\gamma)$. It can be shown that $g'(\gamma)=\gamma\left(\frac{1-q^2(1-\gamma)^2}{4\gamma(1-\gamma)^2(1-q)}-1\right)>0$ in the relevant range $\frac{1-q}{4-q} \leq \gamma \leq \frac{1-q}{2+q}$ and thus $g(\gamma)$ is maximized at $\gamma = \frac{1-q}{2+q}$.

We now examine $h(\gamma)$. We can easily show that $h'(\gamma)=\gamma\left(\frac{(1-q)(2-\gamma)}{9(1-\gamma)^2}-1\right)<0$ in the relevant range $\frac{1-q}{2+q} \leq \gamma \leq \frac{1}{2}$. Therefore, $h(\gamma)$ is maximized at $\gamma = \frac{1-q}{2+q}$.

Finally we examine $j(\gamma)$. Since $j'(\gamma)=(1-q)-\gamma$, $j(\gamma)$ is maximized at $\gamma = 1-q$ if $0<q \leq 0.5$, and is maximized at $\gamma = \frac{1}{2}$ if $0.5 \leq q < 1$.

Combining the above results, we can conclude that (i) the optimal level of network externality is $\gamma = \frac{1-q}{2+q}$ if $0.5 \leq q < 1$ and (ii) if $0 < q \leq 0.5$, to find the optimal level of network externality, we need to compare the high quality firm's net profits when $\gamma = \frac{1-q}{2+q}$ and when $\gamma = 1-q$. Straightforward calculation yields that

$$h\left(\frac{1-q}{2+q}\right)-j(1-q)=\frac{(1-q)\left(-1+q+13q^2+9q^3+2q^4\right)}{2(1+2q)(2+q)^2}\begin{cases}<0 & \text{when } 0<q<0.226 \\ >0 & \text{when } 0.226<q\leq 0.5\end{cases}.$$

We thus have the following proposition.

Proposition 1: *Under low cost environment with k=1, the optimal level of network externality chosen by the high quality firm is*

$$\gamma^* = \begin{cases} 1-q & \text{if } q \le q^c(k) = 0.226 \\ \dfrac{1-q}{2+q} & \text{if } q \ge q^c(k) = 0.226 \end{cases}$$

The market is always fully covered and the market structure can be monopoly or duopoly.

When q is relatively small (as defined above), the high quality firm is a monopolist and chooses a higher level of network externality (i.e. high level of γ); whereas when q is relatively big (as defined above) both the high quality and the low quality firm share the market (duopoly), and the high quality firm chooses a lower level of network externality. The intuition of the result is as follows. When the market is fully covered and monopoly, from the corresponding second stage profit expression ($(1-q)\gamma$), it is not difficult to see that the high quality would like to choose a relatively high value of γ that maximizes its profit. On the other hand, when it has to share the market and the low quality firm gets the benefit from the network (free riding effect), the high quality firm wants to choose a lower value of γ to reduce this negative impact of free riding and keep its competitive advantage. Under the low cost environment, since the market is always fully covered and no scope of growth, keeping the respective market shares is the driving force.

Figure 1 plots the optimal level of network externality as a function of the quality index of the low quality product. We find that the optimal level of network externality is decreasing in q as it should be as the high quality firm wants to limit the free riding effect.

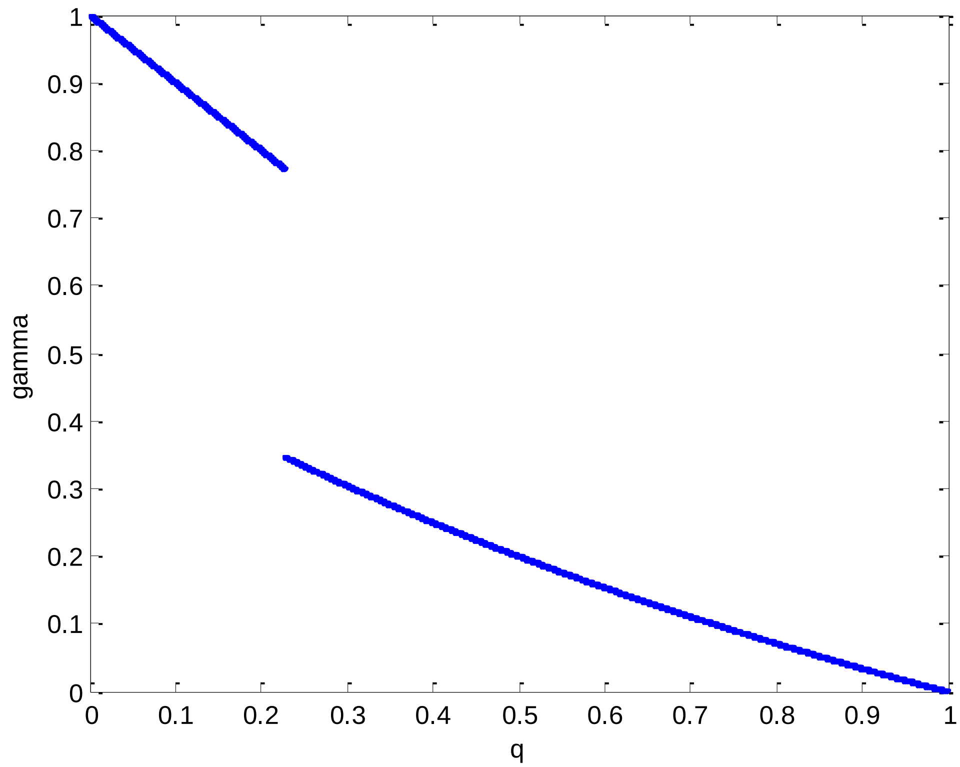

Figure 1 The optimal level of network externality as a function of *q* when *k=1*

5.2 High Cost Environment: *k*=2

Consider $f(\gamma)$ first. By examining $f'(\gamma)$ and $f''(\gamma)$, we can easily show that $f(\gamma)$ is maximized at the interior of the range $\left[0, \frac{1-q}{4-q}\right]$, denoted by γ_1^*. However, a closed-form expression of γ_1^* cannot be derived.

Next we examine $g(\gamma)$. By examining $g'(\gamma)$ and $g''(\gamma)$ and tedious calculation with the help of simulation,[7] we can show that $g(\gamma)$ is maximized at $\gamma = \frac{1-q}{4-q}$ when $0 < q < 0.0917$, at

[7] In the relevant range $\left[\frac{1-q}{4-q}, \frac{1-q}{2+q}\right]$, the function $g(\gamma)$ is monotonically decreasing when $0 < q < 0.0917$, increasing when $0.2619 < q < 1$, while it is increasing first, then decreasing and then increasing again when $0.0917 < q < 0.2619$. Given the complexity of the functional form, we have to turn to simulation when $0.0917 < q < 0.2619$ to determine the optimal level of network externality in this relevant range. Simulation results are available upon request.

the interior of the range $\left[\frac{1-q}{4-q},\frac{1-q}{2+q}\right]$ denoted by γ_2^* when $0.0917<q<0.1612$, and at $\gamma=\frac{1-q}{2+q}$ when $0.1612<q<1$. However, a closed-form expression of γ_2^* cannot be derived.

We now examine $h(\gamma)$. We can easily show that $h'(\gamma)=\gamma\left(\frac{(1-q)(2-\gamma)}{9(1-\gamma)^2}-2\right)<0$ in the relevant range $\frac{1-q}{2+q}\leq\gamma\leq\frac{1}{2}$. Therefore, $h(\gamma)$ is maximized at $\gamma=\frac{1-q}{2+q}$.

Finally we examine $j(\gamma)$. Since $j'(\gamma)=(1-q)-2\gamma<0$, $j(\gamma)$ is maximized at $\gamma=\frac{1}{2}$.

Combining the above results, we can conclude that (i) the optimal level of network externality is γ_1^* if $0<q<0.0917$, (ii) if $0.0917<q<0.1612$, to find the optimal level of network externality, we need to compare the high quality firm's net profits when $\gamma=\gamma_1^*$ and when $\gamma=\gamma_2^*$, and (iii) if $0.1612<q<1$, to find the optimal level of network externality, we need to compare the high quality firm's net profits when $\gamma=\gamma_1^*$ and when $\gamma=\frac{1-q}{2+q}$.

Since the closed-form expressions of γ_1^* and γ_2^* cannot be derived, we have to turn to simulation to find the optimal level of network externality. Simulation yields (1) γ_2^* is always dominated by γ_1^*, i.e., $g(\gamma_2^*)<f(\gamma_1^*)$, and (2) $g\left(\frac{1-q}{2+q}\right)<f(\gamma_1^*)$ when $0.1612<q<0.1644$, while $g\left(\frac{1-q}{2+q}\right)>f(\gamma_1^*)$ when $0.1644<q<1$. Hence, we have the following result.

Proposition 2: *Under high cost environment with k=2, the optimal level of network externality chosen by the high quality firm is*

$$\gamma^*=\begin{cases}\gamma_1^* & \text{if } q\leq q^c(k)=0.1644\\ \frac{1-q}{2+q} & \text{if } q\geq q^c(k)=0.1644\end{cases}$$

The market structure is always duopoly and it can be partially or fully covered.

When q is small, the high quality firm chooses a low level of network externality γ_1^* and the market is partially covered; when q becomes bigger, the high quality firm chooses a higher level of network externality and the market is fully covered. The intuition of the result is as follows. When q is small, the market is not very competitive, hence the high quality firm does not need to choose a high level of γ, given the choice of γ is costly. Whereas, when q gets bigger and the market becomes more competitive, the high quality firm would like to choose a higher level of γ, because of the opportunity of growth in the market (note here the market coverage moves from partial to full) ignoring some of the negative impact of free riding of the low quality firm. In other words, the positive impact of revenue growth dominates the negative impact of free riding for the high quality firm here. This is also evident from the figure below (Figure 2) as we see a big *upward* jump in optimal gamma, when q crosses the low threshold. Thus, although the optimal level of network externality is still decreasing in q piecewise, the overall relationship between the two becomes non-monotonic.

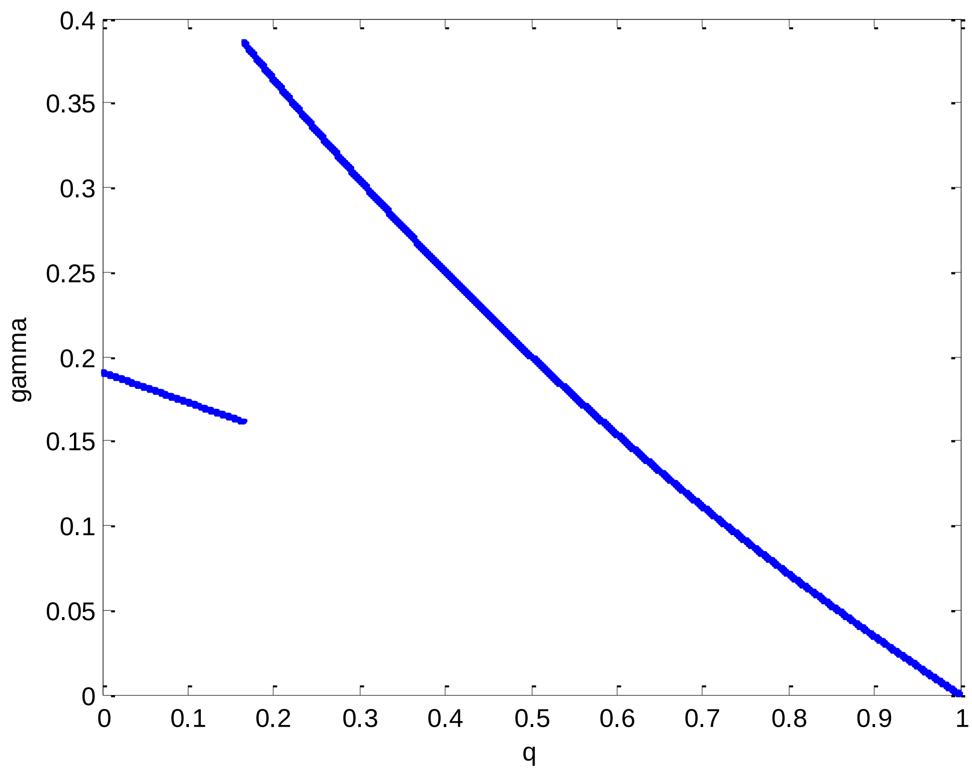

Figure 2 The optimal level of network externality as a function of *q* when *k=2*

6. Optimal Level of Network Externality under All Cost Environments

In this section, we go for a general analysis and find the optimal degree of network externality chosen by the high quality firm under all possible cost environments. Since it is impossible to derive a closed-form expression of the optimal level of network externality for general values of *k* and *q*, we turn to simulation with the help of numerical examples.

Like before, our starting point is high quality firm's net profit:

$$\pi^N(\gamma)=\begin{cases}\dfrac{4(1-q)}{(1-\gamma)(4-q)^2}-\dfrac{1}{2}k\gamma^2 & \text{if } \gamma\le\dfrac{1-q}{4-q},\\ \dfrac{((1-q)+q\gamma)^2}{4(1-q)(1-\gamma)}-\dfrac{1}{2}k\gamma^2 & \text{if } \dfrac{1-q}{4-q}\le\gamma\le\dfrac{1-q}{2+q},\\ \dfrac{(1-q)(2-\gamma)^2}{9(1-\gamma)}-\dfrac{1}{2}k\gamma^2 & \text{if } \dfrac{1-q}{2+q}\le\gamma\le\dfrac{1}{2},\\ (1-q)\gamma-\dfrac{1}{2}k\gamma^2 & \text{if } \gamma\ge\dfrac{1}{2}.\end{cases}$$

Let us refer the above four ranges of γ as the first, second, third, and fourth range respectively.

To find the optimal level of network externality γ^*, we need to find the maximum of $f(\gamma)$, $g(\gamma)$, $h(\gamma)$ and $j(\gamma)$ (all of them as defined before in section 5) in the relevant ranges of γ.

From more numerical analysis, we find that there are three critical values of *k*, k_1^c (in between *1.5* and *1.8*), k_2^c (in between *1.8* and *2*), k_3^c (in between *2* and *6*), which separate *k* into four ranges, each corresponding to one pattern of the range distribution of γ^*.[8] When $k<k_1^c$, as *q* increases from close to 0 to close to 1, γ^* moves from the fourth range to the critical value separating the second range and the third range, i.e., $\dfrac{1-q}{2+q}$. When $k_1^c<k<k_2^c$, as *q* increases from close to 0 to close to 1, γ^* moves from the fourth range to the first range and then to $\dfrac{1-q}{2+q}$. When $k_2^c<k<k_3^c$, as *q* increases from close to 0 to close to 1, γ^* moves from the first range to $\dfrac{1-q}{2+q}$. When $k>k_3^c$, for any $q\in(0,1)$, γ^* always lies in the first range.

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[8] We perform further numerical analysis for k = 1.5, 1.8 and k = 6 apart from k=1, 2 which we have done in Section 5. The results for the new numerical analysis are summarized in Appendix B. The details are available upon request.

For the comparative static analysis, we find that the optimal level of network externality is decreasing in q when $k < k_1^c$ and $k > k_3^c$, while the relationship is non-monotonic when $k_1^c < k < k_2^c$ and when $k_2^c < k < k_3^c$. Figures 3 and 4 plots the optimal level of network externality as a function of the quality index of the low quality product when k=1.8 and k=6. Note that to clearly show that the optimal level of network externality lies in the fourth range for very small values of q when k=1.8, we also insert a small box (within figure 3) where the maximum q is equal to 0.03.

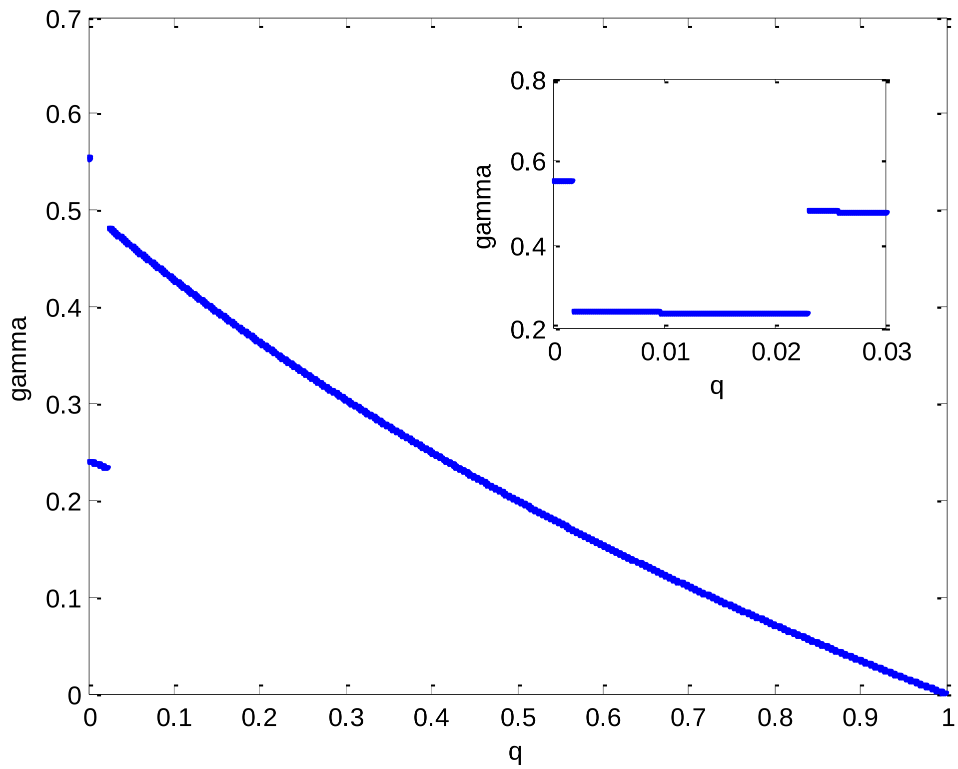

Figure 3 The optimal level of network externality as a function of *q* when *k=1.8*

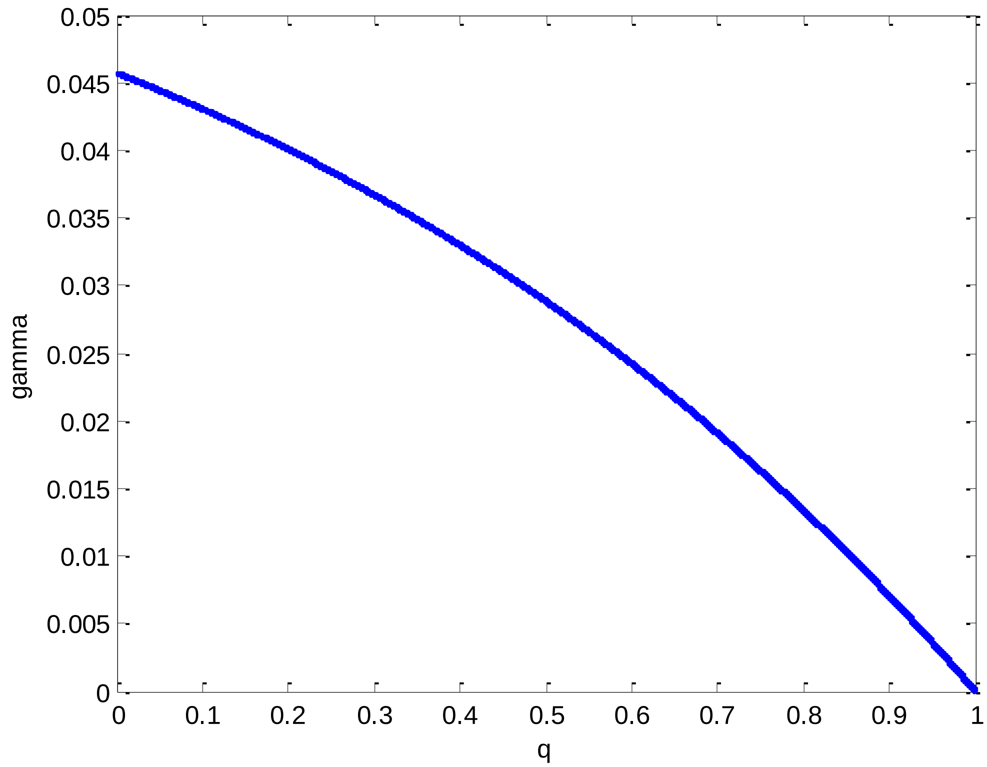

Figure 4 The optimal level of network externality as a function of *q* when *k=6*

Simulation verifies the results obtained from the numerical analysis. Figure 5 presents the main result from the simulation, in which three colour-shaded (blue, green, brown) areas represent the first range, the critical value between the second range and the third range ($\frac{1-q}{2+q}$), and the fourth range respectively. More simulation analysis refines the results and shows that the three critical values of *k* are around 1.784, 1.805 and 5.051.

The interval between *k*=1.784 and *k*=1.805 is very narrow, which makes it difficult to distinguish the second pattern and the third pattern. However, if we look at Figure 5 more closely, we can distinguish these two patterns: The brown-shaded area intersects the *k*-axis at a point above the point at which the brown-, green-, and blue-shaded areas intersect. Alternatively, we can draw another figure to show this difference more clearly. Figure 6 presents the range distribution of γ^* for $1.6 \le k \le 2$ and $0 < q \le 0.1$.

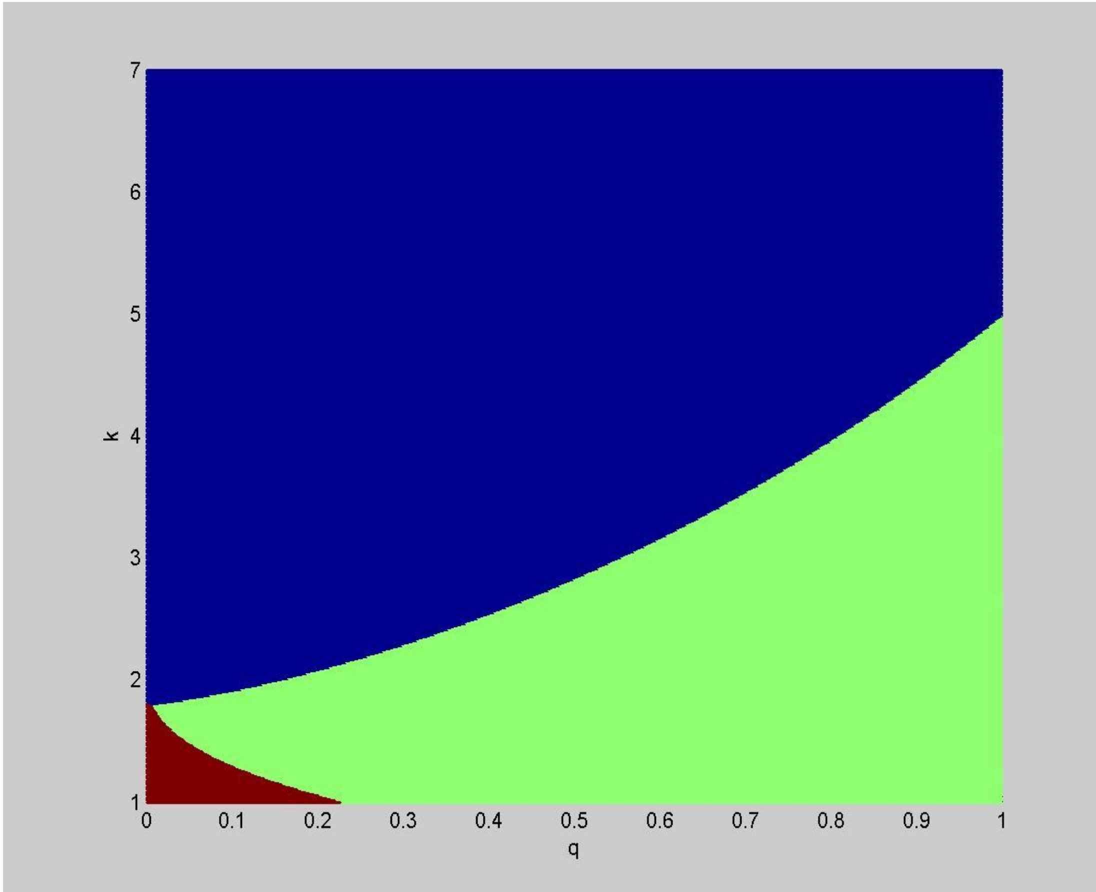

Figure 5 Range distribution of the optimal level of network externality ($1 \le k \le 7$ and $0 < q < 1$**)**

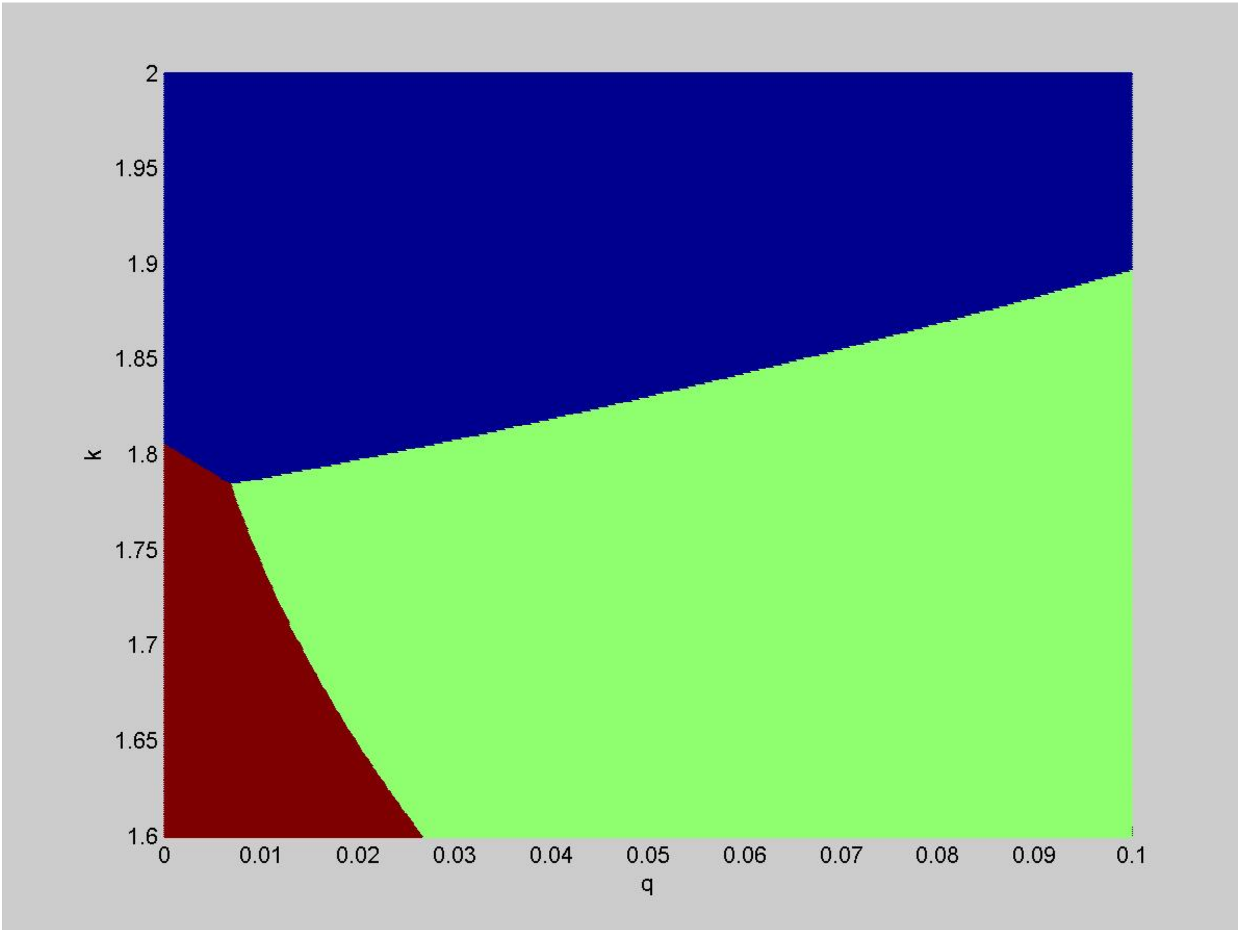

Figure 6 Range distribution of the optimal level of network externality ($1.6 \le k \le 2$ and $0 < q \le 0.1$**)**

From the above analysis, we find that three possible optimal values of γ will actually be chosen by the high quality firm in equilibrium: $\gamma_L^* = \gamma_1^*(q) < \frac{1-q}{4-q}$ (low level of γ), $\gamma_M^* = \frac{1-q}{2+q}$ (medium level of γ), and $\gamma_H^* = \frac{1-q}{k}$ (high level of γ), where $k < k_2^c = 1.805$.

Now corresponding to each optimal value of γ an equilibrium market structure (monopoly or duopoly) is emerged and the market coverage (full or partial) is also determined endogenously.

When the optimal value of γ is γ_L^* which corresponds to a low degree of network externality, the market structure is duopoly i.e. shared by the high quality and the low quality firm and the market is partially covered.

When the optimal value of γ is γ_M^* which corresponds to a medium degree of network externality, the market structure is duopoly i.e. shared by the high quality and the low quality firm and the market is fully covered.

When the optimal value of γ is γ_H^* which corresponds to a high degree of network externality, the market is monopolized by the high quality firm and the market is fully covered.

In the table below, we list all possible equilibrium outcomes, where $q^c(k)$, $q_1^c(k)$ and $q_2^c(k)$ are critical values of q in each relevant case.

Table 1 Equilibrium outcomes for different investment costs and relative quality differences[9]

Investment cost (*k*)	**Low quality Index (*q*)**	**Equilibrium Outcome**		
		Optimal degree of network externality	**Market Structure**	**Market coverage**
Low (k<1.784)	$q \le q^c(k)$	High (γ_H^*)	Monopoly	Full
	$q \ge q^c(k)$	Medium (γ_M^*)	Duopoly	Full
Low-medium (1.784<k<1.805)	$q \le q_1^c(k)$	High (γ_H^*)	Monopoly	Full
	$q_1^c(k) \le q \le q_2^c(k)$	Low (γ_L^*)	Duopoly	Partial
	$q \ge q_2^c(k)$	Medium (γ_M^*)	Duopoly	Full
Medium (1.805<k<5.051)	$q \le q^c(k)$	Low (γ_L^*)	Duopoly	Partial
	$q \ge q^c(k)$	Medium (γ_M^*)	Duopoly	Full
High (k>5.051)	$0 < q < 1$	Low (γ_L^*)	Duopoly	Partial

Below we summarize our main qualitative result from the above analysis.

[9] The relative quality difference is defined to be 1-*q*.

Theorem

In a model of vertical product differentiation with demand network externalities, when the degree of the network externality is a strategic choice of the high quality firm,

(i) the market can be monopolized and fully covered by the high quality firm when the investment cost on the network is low or low-medium and the quality difference between the two products is high, otherwise the market mostly remains duopoly which may be partially or fully covered.

(ii) if the investment cost on the network is very low or very high, the optimal degree of network externality is always increasing in quality difference of the products, but for any other levels of investment cost the relationship is non-monotonic.

7. Conclusion

In the paper, we address issues associated with products with positive network externalities. A firm is not just interested to increase the user-base or network size of its product, it also wants to improve the strength or the degree of network externality to provide more value to its users and gain a strategic advantage over its competitor.[10] We capture this in a model of vertical product differentiation where the high quality firm strategically invests to improve the strength of the network and competes with a low quality firm in the product market. The low quality firms also benefits from the network strength in a limited manner. We solve for the optimal degrees of network externalities for the high quality firm under various network investement costs and characterize endogenous equilibrium market structures and market coverage. In the comparative statics analysis, we show if the investment cost on the network is very low or high, the relationship between the optimal level of network externality and relative quality difference of the products is monotonic, but for any other levels of investment costs, the relationship is non-monotonic.

[10] In the situations, where the products do not have the explicit feature of network externality, the producers as part of their marketing strategy can invest to create a network for the existing and potential users of the product through an interactive digital platform. For example, a firm can invest to create a digital forum of its core products where the existing users can discuss and share their experiences about the products and thereby intensify the effect of network externality of the relevant products. A product developer can also pay to use a popular social networking site, (say like Facebook) to promote its product as this would engage the potential buyers to communicate more effectively and seamlessly about the product through that platform.

The other aspect we address here is, typically in the models of product differentiation with network externality, the degree or the strength of the network effect is mostly assumed as a parameter. We depart from this and endogenize the level of network externality.

We would also like to emphasize the fact that in models of vertical product differentiation more often the extent of the market coverage whether full or partial is assumed to be exogenous. Here we show that covering or not covering the market is at the heart of strategic problem and a consequence of the strategic game between the firms.[11]

We conclude our discussion with a particular scenario where the findings of this analytical model can be useful. These days piracy of digital goods became quite prevalent due to the easy availability of copying technology. It is understood that rampant piracy of digital goods is not desirable from the innovators' as well as society's point of view. In this context, consider our high quality firm as the original firm or the copyright holder of the concerned product and the low quality firm as the commercial pirate. Then our model will suggest that increasing the degree of network effect strategically can eliminate the pirate from the market in some situations (see Table 1). Thus, in the markets where commercial piracy is a serious threat, we propose piracy can be fought by influencing the demand network externality as an effective instrument apart from other regular instruments (like monitoring and/or imposing lump-sum fine to the pirate which are already discussed in detail in the literature of stopping piracy). We think this is an additional instrument, which can be used by the copyright holder or monitoring authority to effectively limit or stop rampant piracy.

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[11] In another rare instance, Wauthy (1996) considered a two stage duopoly game of quality-price competition and emphasized the endogenous aspect of market coverage.

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Appendix A

The high-quality producer maximizes $p_H D_H = \frac{1}{1-\gamma} p_H \left(1 - \frac{p_H - p_L}{1-q}\right)$. The first-order condition is

$$1 - q - 2p_H + p_L = 0. \tag{A1}$$

The low-quality producer maximizes

$p_L D_L = \frac{1}{1-\gamma} p_L \left(\frac{p_H - p_L}{1-q} - \gamma \right) = \frac{1}{(1-\gamma)(1-q)} p_L \left(p_H - p_L - \gamma(1-q) \right)$ subject to $q\gamma - p_L \geq 0$. Define the Lagrangian $L = p_L \left(p_H - p_L - \gamma(1-q) \right) + \lambda \left(q\gamma - p_L \right)$. The first-order and slackness conditions are

$$\begin{gathered} \frac{\partial L}{\partial p_L} = p_H - 2p_L - \gamma(1-q) - \lambda = 0, \\ \lambda \geq 0,\ q\gamma - p_L \geq 0,\ \lambda\left(q\gamma - p_L\right) = 0. \end{gathered} \tag{A2}$$

If $p_L = q\gamma$, then we must have $\lambda = p_H - 2p_L - \gamma(1-q) \geq 0$. Given $p_L = q\gamma$, it follows that

$$p_H = (1 - q + p_L)/2 = (1 - q + q\gamma)/2, \tag{A3}$$

and

$$\lambda = p_H - 2p_L - \gamma(1-q) = (1-q+q\gamma)/2 - 2q\gamma - \gamma(1-q) = \tfrac{1}{2}\left(1 - q - (2+q)\gamma\right).$$

Thus, $\lambda \geq 0 \Leftrightarrow \gamma \leq \dfrac{1-q}{2+q} = \hat{\hat{\gamma}}$

If $q\gamma - p_L \geq 0$, then we must have $\lambda = 0$ and

$$p_H - 2p_L - \gamma(1-q) = 0. \tag{A4}$$

The solution to the system of equations (A.1) and (A.4) is

$$p_H = (1-q)(2-\gamma)/3,\ \ p_L = (1-q)(1-2\gamma)/3. \tag{A5}$$

It follows that

$$q\gamma - p_L = q\gamma - (1-q)(1-2\gamma)/3 = \left(\gamma(2+q) - (1-q)\right)/3.$$

Thus,

$$q\gamma - p_L \geq 0 \Leftrightarrow \gamma \geq \frac{1-q}{2+q} = \hat{\hat{\gamma}}.$$

Note that for (A5) to be the solution, γ must not exceed $1/2 : \gamma \leq \tilde{\gamma} = \dfrac{1}{2}$; so that the price of the low-quality product is nonnegative: $p_L \geq 0$.

The demands and profits for both firms are then obtained straightforwardly. The expressions are given in Section 4.1 and will not be repeated here.

Appendix B

Summary of the results for k = 1.5, 1.8 and 6. Details are available upon request.

When *k*=1.5, the optimal level of network externality chosen by the high quality firm is

$$\gamma^* = \begin{cases} \dfrac{2(1-q)}{3} & \text{if } q \le q^c(k) = 0.0444 \\ \dfrac{1-q}{2+q} & \text{if } q \ge q^c(k) = 0.0444 \end{cases}.$$

When *k*=1.8, the optimal level of network externality chosen by the high quality firm is

$$\gamma^* = \begin{cases} \dfrac{5(1-q)}{9} & \text{if } q \le q_1^c \approx 0.0017 \\ \gamma_1^*(q) < \dfrac{1-q}{4-q} & \text{if } q_1^c \le q \le q_2^c \\ \dfrac{1-q}{2+q} & \text{if } q \ge q_2^c \approx 0.023 \end{cases}.$$

When *k*=6, the optimal level of network externality chosen by the high quality firm is $\gamma^* = \gamma_1^*(q) < \dfrac{1-q}{4-q}$.