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Abstract

Based on the multifractal detrended fluctuation analysis (MF-DFA) and multifractal spectrum analysis, this paper empirically studies the multifractal properties of the Chinese stock index futures market. Using a total of 2,942 ten-minute closing prices, we find that the Chinese stock index futures returns exhibit long-range correlations and multifractality, making the single-scale index insufficient to describe the futures price fluctuations. Further, by comparing the original time series with the transformed time series through shuffling procedure and phase randomization procedure, we show that there exists two different sources of the multifractality for the Chinese stock index futures market. Our results suggest that the multifractality is mainly due to long-range correlations, although the fat-tailed probability distributions also contribute to such multifractal behavior.

Keywords: Multifractality; Stock index futures; MF-DFA; Generalized Hurst exponent

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1. Introduction

Since the launch of stock index futures on the China Financial Futures Exchange (CFFE) in April 2010, China's financial market has become more complete and integrated with the rest of world. The index futures are referenced to the China stock index 300 (CSI300), which consists of 300 RMB-denominated A shares actively traded on the Shanghai and Shenzhen stock exchanges.

This paper aims to empirically test whether returns on the newly established Chinese stock index futures exhibit long-range correlations and multifractal patterns. The term *fractal* was coined by Mandelbrot [1] to characterize a rough or fragmented geometric shape that displays a large degree of self similarities within its own fractional dimensions. In recent years, fractal patterns have been extensively studied in diverse fields, ranging from physics to economics and finance.

In the literature, several approaches have been developed and applied to the exploration of fractal properties. For instance, the rescaled adjusted range analysis (R/S analysis) was introduced by Hurst [2] for his hydrological study. The well-known Hurst exponent, which is directly related to the fractal dimension, has now been widely used in the analyses of financial market volatility. Due to the difficulty of R/S analysis in capturing long-range correlations of non-stationary series, Peng et al. [3] proposed the detrended fluctuation analysis (DFA) method in order to analyse DNA sequences. Although DFA has become a widely used approach for the determination of monofractal scaling properties, it can not be applied to describe the multi-scale and fractal subsets of the time series. Based on a generalization of DFA, Kantelhardt et al. [4] advanced the multifractal detrended fluctuation analysis (MF-DFA) for the multifractal characterization of non-stationary time series.

As a robust and powerful technique for the verification of multifractal behavior, MF-DFA has so far been applied to various markets, including international crude oil markets [5], foreign exchange markets [6], stock markets [7], gold markets [8], and agricultural commodity futures markets [9]. Scholars have also studied the cross-correlations between two non-stationary time series [10-13] by generalising DFA and MF-DFA analyses with an emphasis on detrended covariance. However, none of the existing studies has applied the MF-DFA approach to the stock index futures market, in particular in the context of Chinese economy.

Using 10-minute closing prices of the CSI300 futures contract, this paper enriches the MF-DFA literature by analyzing the characteristics of China's stock index futures market. The purpose is

two-fold: First, to demonstrate that returns to the Chinese stock index futures exhibit long-range correlations and multifractal patterns; second, to discuss the sources of such multifractality in the futures market. To our best knowledge, this study represents the first attempt to explore the multifractal properties of the Chinese stock index futures returns.

Research in the area of stock index futures has largely focused on well-developed markets [14-21], while emerging futures markets have not yet received much attention. In particular, very limited research was previously done on the emerging stock index futures market in China. Until quite recently, Wen et al. [22] examined the market efficiency and hedging effectiveness of the Chinese index futures market, and Yang et al. [23] investigated the intruder price discovery and volatility transmission between the Chinese CSI300 index market and the stock index futures market. Nonetheless, none of the prior researches has tackled the multifractality and its possible sources for the Chinese stock index futures market.

The remainder of the paper is organized as follows. Section 2 specifies the MF-DFA methodology used for this study. Section 3 describes our data set. Section 4 presents the empirical results. Section 5 is reserved for the conclusion.

2. Methodology

Let $\{x_k, k = 1, ..., N\}$ be a time series, where N is the length of the series. The MF-DFA procedure consists of the following steps:

Step 1: Determine the profile

$$Y_i = \sum_{k=1}^{i} (x_k - \bar{x}), i = 1, 2, ..., N \tag{1}$$

where $\bar{x}$, $\bar{x} = \dfrac{1}{N} \sum_{k=1}^{N} x_k$, denotes the averaging over the whole time series.

Step 2: Divide the profile Y_i into $N_s = \text{int}\left(\dfrac{N}{S}\right)$ non-overlapping segments according to the length of s. Since the length N is often not a multiple of the considered time scale s, the same

dividing procedure is repeated starting from the opposite end in order not to disregard some part of the time series. Thereby, $2N_s$ segments are obtained altogether.

Step 3: Calculate the local trend for each sub-interval υ, where $\upsilon = 1, 2, ..., 2N_s$. Then we get the fitting equation

$$P_v(i) = a_0 + a_1 i + ... a_k i^k, i = 1, 2, ..., s; k = 1, 2, ... \qquad (2)$$

$$\text{where } s \geq \max\{k + 2, 10\}.$$

Here, $k=1,2,...m$ means polynomial with order m is used when making a regression of s time series points[1].

Step 4: Determine the variance by eliminating the local trend of each sub-interval υ.

$$F^2(s,v) = \frac{1}{s}\sum_{i=1}^{s}\left[Y\big((v-1)s+i\big) - P_v(j)\right]^2, v = 1, 2, ..., N_s \qquad (3)$$

or

$$F^2(s,v) = \frac{1}{s}\sum_{i=1}^{s}\left[Y\big(N-(v-N_s)s+i\big) - P_v(j)\right]^2, v = N_s+1, N_s+2, ..., 2N_s \qquad (4)$$

Here, $P_v(j)$ is the fitting polynomial with order m in segment v (conventionally, called mth order MF-DFA and wrote MF-DFAm).

Step 5: Average over all segments to obtain the q-th order fluctuation function.

$$F_q(s) = \left\{\frac{1}{2N_s}\sum_{v=1}^{2N_s}\left[F^2(s,v)\right]^{\frac{q}{2}}\right\}^{\frac{1}{q}} \qquad q \neq 0 \qquad (5)$$

$$F_0(s) = \exp\left\{\frac{1}{4N_s}\sum_{v=1}^{2N_s}\ln\left[F^2(s,v)\right]\right\} \qquad q = 0 \qquad (6)$$

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[1] For a detailed discussion of detrending method with varying polynomial orders, see Horvatic et al. [24]

Step 6: Determine the scaling behaviour of the fluctuation functions by analyzing log-log plot of $F_q(s)$ versus s for each value of q. If the time series x_k is long-range power-law correlated, $F_q(s)$ increases for large value of *s* as a power-law,

$$F_q(s) \propto s^{h(q)} \qquad (7)$$

where h(q) refers to the generalized Hurst exponent.

Step 7: Tue.(7) can be written as $F_q(s) = As^{h(q)}$, we then have, after taking logarithms of both sides:

$$\log F_q(s) = \log A + h(q)\log s \qquad (8)$$

Then the estimated value of Generalized Hurst exponent *h(q)* can be obtained.

For monofractal time series characterized by a single exponent over all time scales, *h(q)* is independent of *q*. For multifractal time series, *h(q)* varies with *q*. The different scaling of small and large fluctuation will yield a significant dependence of *h(q)* on *q*. Therefore, for positive value of *q*, *h(q)* describes the scaling behaviour of the segments with large fluctuations; and for negative q values, the scaling exponent *h(q)* describes the scaling behaviour of segments with small fluctuations.

Another way of confirming multifractality in time series is through multifractal spectrum analysis, which is based on the following relationship between Generalized Hurst exponent h(q) obtained from MF-DFA and the Renyi exponent $\tau(q)$:

$$\tau(q) = qh(q) - 1 \qquad (9)$$

Then, through a Legendre transform, we get:

$$\alpha = h(q) + qh'(q) \qquad (10)$$

$$f(\alpha) = q\left[\alpha - h(q)\right] + 1 \qquad (11)$$

3. Data

The data used for this study are the time series of 10-minute closing prices of the IF1009, over 107 trading days from April 16, 2010 to September 17, 2010. The IF1009 refers to the

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September 2010 index futures contract tied to CSI300, and it is one of the most actively traded contracts in 2010.   Our dataset consists of 2942 observations, which are obtained from Wind@database.

The 10-minute price returns R_t is defined as the logarithmic difference between the following closing prices:

$$R_t = \ln P_{t+1} - \ln P_t, t = 1, 2, ..., N-1 \tag{12}$$

where P_t is the closing price of stock index futures at the business time t.

The 10-minute closing price and normalized log- returns of stock index futures are presented in Fig.1 and Fig.2 respectively.
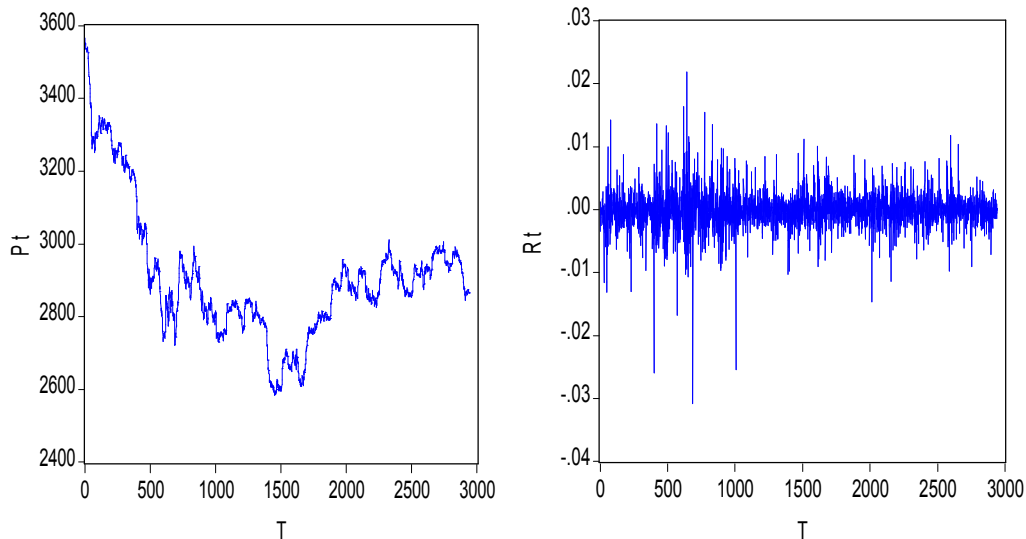

Fig.1 Closing price of Stock Index Futures   **Fig. 2** Returns of Stock Index Futures

4. Empirical Results

4.1 The multifractal characteristics of stock index futures

Fig. 3 shows the log-log plot of fluctuation $F_q(s)$ versus s for various values of q, according to equations (1) to (8). We calculate $F_q(s)$ for time scale s ranging from 3 to $N/5$, where N is the total length of time series. The upper and the lower curves correspond to the cases of q =10 and

q = -10 respectively. It can be seen from Fig. 3 that the slope increases with the decrease in the value of q. Further, the distance between neighbour curves increases first (*q*>0) and then declines (*q*<0), indicating that the value of *h(q)* eventually converges.
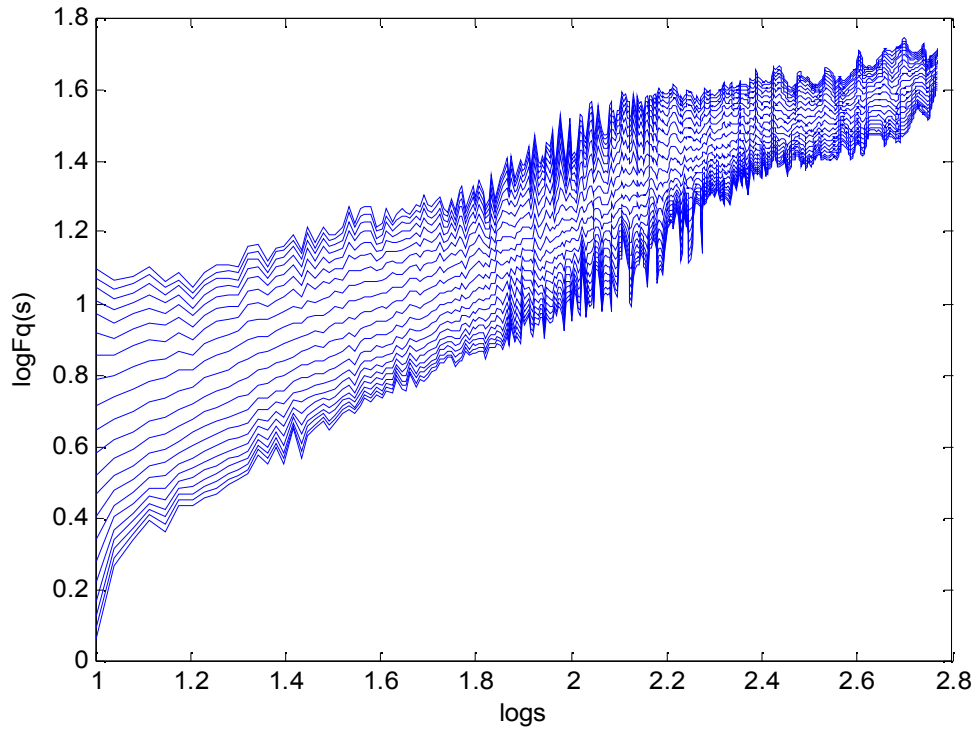

Fig. 3 Log-log plot of $F_q(s)$ curve for time scale *s*

The generalized Hurst exponents *h(q)* for different *q* ranging from -10 to 10 is shown in Table1. When the value of *q* rises from -10 to 10, the generalized Hurst exponents *h(q)* falls from 0.7154 to 0.3880. That is to say, China's stock index futures market possesses the multifractal features, making single fractal models irrelevant for the analysis. Noteworthy is that, when q=2, the generalized Hurst exponent *h(q)* obtained from MF-DFA is identical to the Hurst exponent H in DFA. As *h(2)* = 0.5355 > 0.5, we conclude that the returns of China's stock index futures are strongly persistent.

Taking *h(q)* = 0.5 as the boundary, the generalized Hurst exponent *h(q)* of stock index futures returns decreases as the value of q increases. More specifically, for the values of $q \leq 3$, the generalized Hurst exponents *h(q)* is larger than 0.5. The lower the value of q, the closer is the value of *h(q)* to 1. This indicates that small fluctuations of stock index futures returns are persistent. However, when $q > 3$, the generalized Hurst exponents *h(q)* becomes smaller than 0.5,

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and the value of *h(q)* gets closer to 0 with the increase in *q*. In this case, large fluctuations of stock index futures returns display anti-persistent properties.

Table 1 Generalized Hurst exponent *h(q)* of stock index futures returns

q	h(q)		
	Original Data	Shuffled Data	Surrogate Data
-10	0.7154	0.5863	0.5654
-9	0.7093	0.5794	0.5585
-8	0.7023	0.5717	0.5507
-7	0.6941	0.5631	0.5421
-6	0.6846	0.5536	0.5327
-5	0.6736	0.5435	0.5225
-4	0.6608	0.5332	0.5121
-3	0.6461	0.5231	0.5021
-2	0.6293	0.5139	0.4889
-1	0.6099	0.5059	0.4799
0	0.5877	0.4991	0.4661
1	0.5626	0.4932	0.4602
2	0.5355	0.4879	0.4544
3	0.5079	0.4827	0.4480
4	0.4820	0.4775	0.4446
5	0.4590	0.4721	0.4393
6	0.4394	0.4666	0.4339
7	0.4229	0.4612	0.4224
8	0.4092	0.4560	0.4187
9	0.3976	0.4510	0.4076
10	0.3880	0.4463	0.3992
Δh	0.3274	0.1400	0.1662

Note: The table shows the generalized Hurst exponent *h(q)* when *q* varies from -10 to 10. *h(q)*>0.5: the series displays the persistence; *h(q)*<0.5, the series displays anti-persistence; *h(q)*=0.5, the series displays random walk behavior.

Fig. 4 presents the path of generalized fractal dimension D_q, which is generated from the

multifractal spectrum model based on equations (9) to (11). We can see that D_q decreases with a higher value of q, and the returns on stock index futures show strong multifractality.
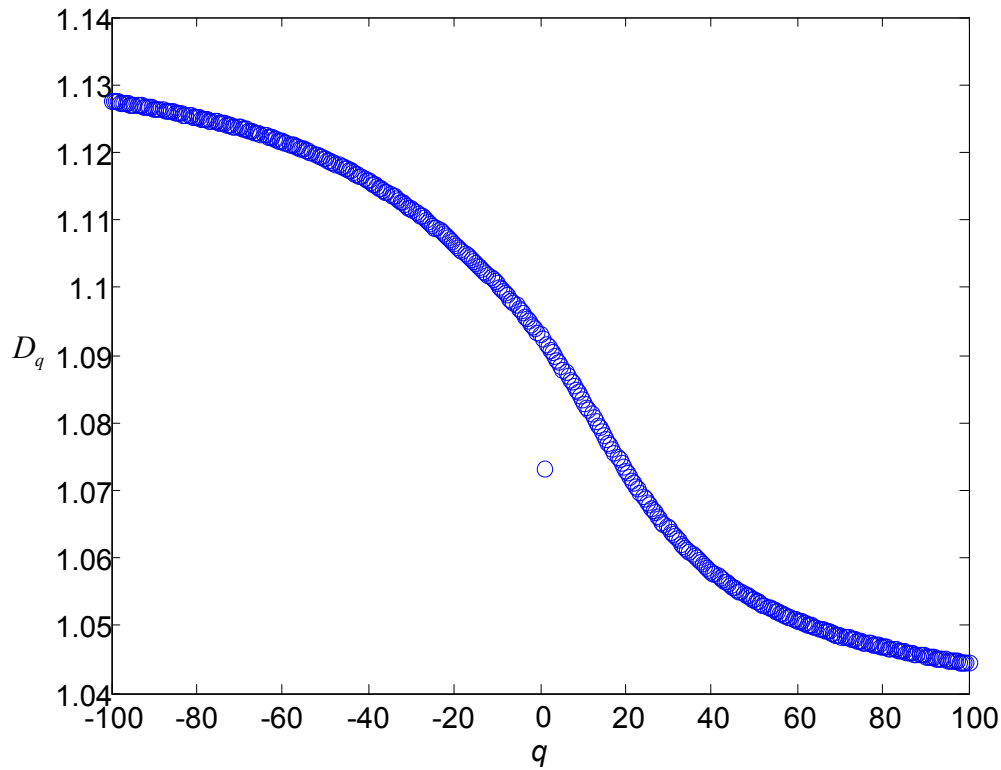

Fig. 4 The path of generalized fractal dimension D_q

4.2 The sources of multifractality

Generally speaking, there are two factors contributing to multifractal properties, namely long-range temporal correlations for small and large fluctuations and the fat-tailed probability distributions of variations. To investigate the sources of multifractality of China's stock index futures returns, we calculate both the generalized Hurst exponents for original, shuffled and surrogate returns of stock index futures (see Table 1 and Fig. 5) and the corresponding multifractality degrees (see Table 2). It may be noted that shuffling procedure destroys any temporal correlations while preserving the distribution of fluctuations, and the phase randomization procedure weakens non-Gaussianity in time series.

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Therefore, a comparison between the generalized Hurst exponents and its fluctuation ranges of original time series and transformed time series enables us to find the sources of multifractality for China's stock index futures market.

According to Table 1 and Fig. 5, the extent to which $h(q)$ varies with the value of q is much less for transformed times series than original data of stock index futures returns, while the degree of multifractality appears to be weaker for transformed time series than original time series. This suggests that both long-range correlations and fat-tailed distributions contribute to the multifractality pattern of the stock index futures returns in China.

In terms of the transformed time series, it is evident that surrogate time series exhibit relative less multifractality than shuffled time series (i.e. $\Delta h_{su} \succ \Delta h_{sh}$ in Table 1). The implication is that long-range temporal correlations for small and large fluctuations exert greater influences on the multifractality of China's stock index futures market.
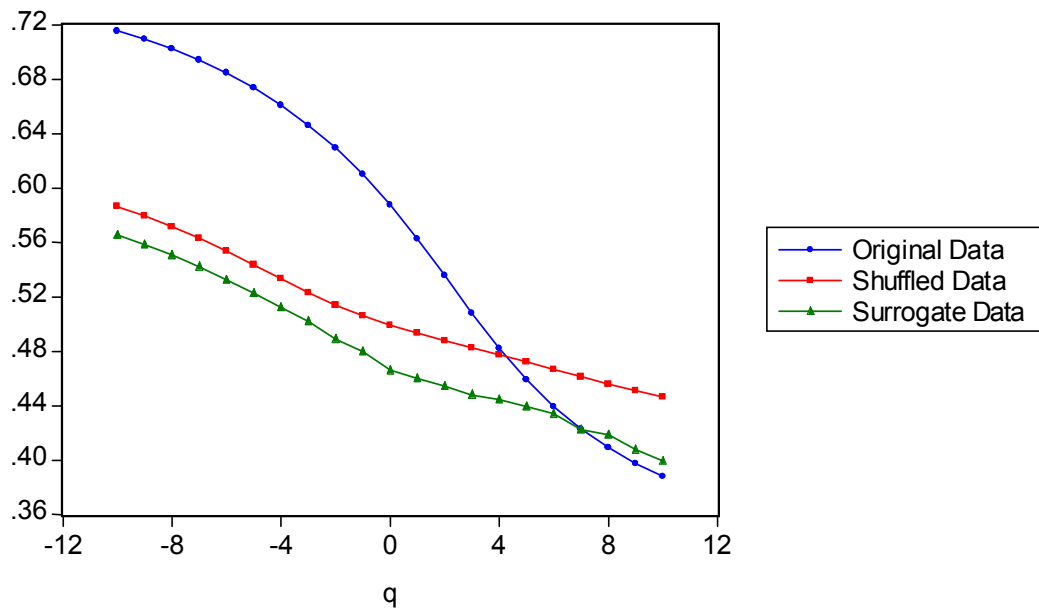

Fig. 5 Generalized Hurst exponents *h(q)* for different time series

To gain a further understanding of the strength of multifractality of stock index futures returns, the width of fractal spectrum $\Delta\alpha$ is derived for each time series using equations (9) to (11). The results presented in Table 2 show that $\Delta\alpha$ in the transformed series is much smaller than in the original series, confirming that the persistence correlation plays a more

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important role in the multiscaling of the returns variations than fat-tailed distributions. Further, the spectrum width of shuffled data is narrower than that of surrogate data, and this can be attributable to the impact of extremely large non-Gaussian events on the multifractality patterns.

Table 2 Multifractality degrees of China's stock index futures returns

	α_{max}	α_{min}	$\Delta\alpha$
Original Data	1.1373	1.0348	0.1026
Shuffled Data	0.9274	0.8401	0.0873
Surrogate Data	0.8637	0.7645	0.0992

Note: α, as a singular index, is used to describe singular degree of different intervals in complex system. The larger the value of α, the smaller is the degree of singularity.

$\Delta\alpha$ is the width of fractal spectrum, which shows the distinction between the maximum probability and the minimum probability (i.e. $\Delta\alpha = \alpha_{max} - \alpha_{min}$). The larger the value of $\Delta\alpha$, the more uneven is the distribution of time series, and thus the stronger is the multifractality.

5. Conclusion

Using the multifractal detrended fluctuation analysis (MF-DFA) together with multifractal spectrum analysis, this paper explores multifractal properties of the Chinese stock index futures returns. We find that stock index futures market in China exhibits multifractal patterns, making the single-scale exponent inappropriate for analyzing the futures returns. Indeed, the multifractal detrended fluctuation analysis (MF-DFA) leads to a much better understanding of the complex financial futures market.

Further, through comparing Δh and $\Delta\alpha$ for original series and transformed series, we verify that the multifractality degree of stock index futures returns is mostly due to different long-range correlations for small and large fluctuations. The fat-tailed distributions, to a certain extent, also contribute to the multifractal behaviour of the time series.

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