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Patent licensing in spatial models

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Patent Licensing in Spatial Models

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Abstract

We show that a two-part tariff licensing contract is always optimal to the insider patentee in spatial models irrespective of the size of the innovation or any pre-innovation cost asymmetries. The result provides a simple justification of the prevalence of two-part tariff licensing contracts in industries.

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Key Words: Salop Model, Hotelling Model, Costs, Innovation, Patent Licensing

1 Introduction

Patent licensing is a fairly common practice that takes place in almost all industries. It is a source of profit for the innovator (also called licensor or patentee) who earns rent from the licensee by transferring a new technology using various licensing contracts. Among them a two-part tariff licensing contract is widely observed in reality. Typically, in a two-part tariff contract there is fixed component and a variable component. The fixed component can be determined by a simple fixed fee or auction (depending on the number of licensees) and the variable part is determined by using per-unit or ad valorem royalty. Rostoker (1983) in an empirical work finds that royalty payments alone are used in 39% of the cases, a fixed fee alone in 13%, and both instruments together in 46%. Taylor and Silberston (1973) find similar percentages in their study. More recently, Macho-Stadler et al. (1996) find, using Spanish data, that 59% of the contracts have royalty payments, 28% fixed fee payments, and 13% include both fixed and royalty fees.

In this paper, we show why a two-part tariff licensing consisting of a fixed fee and a per-unit royalty can be a dominant mode of licensing in the industries. We prove that in spatial models of competition with an insider patentee, the optimal licensing contract is always a two-part tariff scheme. Specifically, we

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show this result in Salop's circular city model and Hotelling's linear city model, the two most celebrated models in spatial competition in economics. Our result is robust to all possible innovations i.e. drastic or non-drastic; and all possible pre-innovation cost asymmetries between the patentee and licensee. Thus we provide a simple justification for the prevalence of two-part tariff licensing contracts.

There is a vast literature (see Kamien (1992) for a survey on patent licensing, and Sen and Tauman (2007) for general licensing schemes), which focuses on the optimal licensing arrangement by the patentee in a wide variety of situations. Interestingly, all these studies are done in a standard framework of price and/or quantity competition (i.e. the representative consumer approach of product differentiation) but very few studies are done in spatial framework.[1] We believe that the spatial models, like Salop and Hotelling, are an appropriate place to study the licensing behaviour of firms in the industries where markets are already developed and not growing over time while the differentiation over the brands is well established and is not changing rapidly. In a typical location model, when the full market is always served, the quantity demanded at each price not sufficiently high does not change. We believe this particular feature in a location model is important, when one compares across equilibrium outcomes (equilibrium prices, profits of the firms) under different licensing regimes as the market size (or aggregate demand) remains constant across the regimes.

The rest of the paper is organized as follows. Licensing in Salop's model is discussed in section 2. Licensing in Hotelling's model is discussed in section 3. Section 4 concludes with few remarks.

2 Salop's model

Consider a circular city with unit circumference. Two firms produce a homogeneous good, located symmetrically on the city and compete in prices. Suppose firm A is located at 0 and firm B is located at 1/2. Consumers are uniformly distributed over the circular rim and buys exactly one unit of the good either from firm A or B. The transportation cost per unit of distance is t. The utility function of a consumer located at x and buying from firm A is

$$\begin{aligned} U_A &= v - p_A - tx && \text{if } x \in [0, 1/2], \\ &= v - p_A - t(1-x) && \text{if } x \in [1/2, 1]. \end{aligned}$$

The utility function of a consumer located at x and buying form firm B is

$$\begin{aligned} U_B &= v - p_B - t(1/2 - x) && \text{if } x \in [0, 1/2], \\ &= v - p_B - t(x - 1/2) && \text{if } x \in [1/2, 1]. \end{aligned}$$

[1] See Caballero et. al. (2002); Poddar & Sinha (2004); and Matsumura & Matsushima (2008).

Assume the market is fully covered. It is straightforward to derive the demand for firms A and B, which is given below:

$$
\begin{aligned}
Q_A &= \frac{1}{2} + \frac{p_B - p_A}{t} && \text{if } p_B - p_A \in \left[-\frac{t}{2}, \frac{t}{2}\right], \\
&= 0 && \text{if } p_B - p_A < -\frac{t}{2}, \\
&= 1 && \text{if } p_B - p_A > \frac{t}{2},
\end{aligned}
$$

and

$$Q_B = 1 - Q_A.$$

2.1 Pre-innovation

Denote the marginal costs of production of A and B by c_A and c_B respectively and define $\delta = c_A - c_B$. We need to assume $-\frac{3t}{2} < \delta < \frac{3t}{2}$ so that the less efficient firm's equilibrium quantity is positive before the innovation takes place. The equilibrium prices, demands and profits are given by the following:

$$
\begin{aligned}
p_A &= \frac{1}{6}(3t + 4c_A + 2c_B) = c_A + \frac{1}{6}(3t - 2\delta), && (1) \\
p_B &= \frac{1}{6}(3t + 2c_A + 4c_B) = c_B + \frac{1}{6}(3t + 2\delta), && (2) \\
Q_A &= \frac{1}{6t}(3t - 2\delta), && (3) \\
Q_B &= \frac{1}{6t}(3t + 2\delta), && (4) \\
\pi_A &= \frac{1}{36t}(3t - 2\delta)^2, && (5) \\
\pi_B &= \frac{1}{36t}(3t + 2\delta)^2. && (6)
\end{aligned}
$$

Suppose firm A is the innovative firm (also called patentee) and it comes up with a cost reducing innovation which reduces the marginal cost of production by $\varepsilon \in (0, \min\{c_A, c_B\}]$. ε measures the size of innovation. Assume firm B is the potential licensee. The upper limit of ε ensures no firm's marginal cost becomes negative after the innovation and the licensing.

A licensing game consists of three stages. In the first stage, the patent holding firm A decides whether to provide license or not to firm B for the new technology, and if decides to provide then it sets the licensing contract. In the second stage, firm B decides whether to accept or reject the offer from firm A. In the final stage, both firms compete in prices.

Below, we list the equilibrium outcomes under all the relevant scenarios that we need to consider for our analysis.

2.2 No licensing

Consider the no licensing case first. Substituting c_A by $c_A - \varepsilon$ into (1)-(6),[2] we get

$$
\begin{aligned}
p_A^{NL} &= c_A - \varepsilon + \frac{1}{6}\left(3t + 2\left(\varepsilon - \delta\right)\right), &(7)\\
p_B^{NL} &= c_B + \frac{1}{6}\left(3t - 2\left(\varepsilon - \delta\right)\right), &(8)\\
Q_A^{NL} &= \frac{1}{6t}\left(3t + 2\left(\varepsilon - \delta\right)\right), &(9)\\
Q_B^{NL} &= \frac{1}{6t}\left(3t - 2\left(\varepsilon - \delta\right)\right), &(10)\\
\pi_A^{NL} &= \frac{1}{36t}\left(3t + 2\left(\varepsilon - \delta\right)\right)^2, &(11)\\
\pi_B^{NL} &= \frac{1}{36t}\left(3t - 2\left(\varepsilon - \delta\right)\right)^2, &(12)
\end{aligned}
$$

where the superscript NL denotes no licensing case.

Clearly, the equilibrium outcome is as described above only when $\varepsilon < \delta + \frac{3t}{2}$, i.e., when the innovation is non-drastic. The innovation is drastic when $\varepsilon \geq \delta + \frac{3t}{2}$, i.e., when the quantity demanded of firm B's product is zero even if its price is equal to its marginal cost c_B. In this case, $p_A^{NL} = c_B - \frac{t}{2}$, $\pi_A^{NL} = c_B - \frac{t}{2} - (c_A - \varepsilon) = \varepsilon - \delta - \frac{t}{2}$, $\pi_B^{NL} = 0$.

2.3 Fixed Fee Licensing

Now consider the fixed fee licensing. Let F denote the fixed fee. The patentee will set the fixed fee such that firm B is indifferent between accepting the licensing contract and not accepting the contract. If firm B accepts the contract, then firm B's marginal cost becomes $c_B - \varepsilon$. Substituting c_A by $c_A - \varepsilon$ and c_B by $c_B - \varepsilon$ into (1)-(6) and also noticing there is a monetary transfer F from firm B to firm A, we get

$$
\begin{aligned}
p_A^{F} &= c_A - \varepsilon + \frac{1}{6}\left(3t - 2\delta\right), &(13)\\
p_B^{F} &= c_B - \varepsilon + \frac{1}{6}\left(3t + 2\delta\right), &(14)\\
Q_A^{F} &= \frac{1}{6t}\left(3t - 2\delta\right), &(15)\\
Q_B^{F} &= \frac{1}{6t}\left(3t + 2\delta\right), &(16)\\
\pi_A^{F} &= \frac{1}{36t}\left(3t - 2\delta\right)^2 + F, &(17)\\
\pi_B^{F} &= \frac{1}{36t}\left(3t + 2\delta\right)^2 - F, &(18)
\end{aligned}
$$

[2] Note that if we substitute c_A by $c_A - \varepsilon$ into expressions involving δ, then we should also substitute δ by $\delta - \varepsilon$ since the cost difference between the two firms changes from δ to $\delta - \varepsilon$.

where the superscript F denotes fixed fee licensing case. So the patentee will set the fixed fee equal to $\frac{1}{36t}(3t+2\delta)^2 - \pi_B^{NL}$.

When the innovation is nondrastic, $F = \frac{1}{36t}(3t+2\delta)^2 - \frac{1}{36t}(3t-2(\varepsilon-\delta))^2 = \frac{\varepsilon}{9t}(3t+2\delta-\varepsilon) > 0$ since $0 < \varepsilon < \delta + \frac{3t}{2}$. Firm A's profit is then $\pi_A^F = \frac{1}{36t}(3t-2\delta)^2 + F = \frac{1}{36t}(3t-2\delta)^2 + \frac{\varepsilon}{9t}(3t+2\delta-\varepsilon) = \frac{1}{36t}\left((3t-2\delta)^2 + 4\varepsilon(3t+2\delta-\varepsilon)\right)$. When the innovation is drastic, $F = \frac{1}{36t}(3t+2\delta)^2$. Firm A's profit is $\pi_A^F = \frac{1}{36t}(3t-2\delta)^2 + \frac{1}{36t}(3t+2\delta)^2 = \frac{1}{18t}\left(4\delta^2 + 9t^2\right)$.

Now we compare firm A's profit in the fixed fee licensing case and in the no licensing case. When the innovation is nondrastic, $\pi_A^{NL} - \pi_A^F = \frac{1}{36t}(3t+2(\varepsilon-\delta))^2 - \frac{1}{36t}\left((3t-2\delta)^2 + 4\varepsilon(3t+2\delta-\varepsilon)\right) = \frac{2}{9t}\varepsilon(\varepsilon-2\delta)$; When the innovation is drastic, $\pi_A^{NL} - \pi_A^F = \varepsilon - \delta - \frac{t}{2} - \frac{1}{18t}\left(4\delta^2+9t^2\right) = -\frac{1}{9t}\left(9t\delta - 9t\varepsilon + 2\delta^2 + 9t^2\right) > 0$.[3] Hence, we have the following result.

Lemma 1 *In Salop's circular city model, no licensing is always better than fixed fee licensing for the patentee except that when the patentee is inefficient ($\delta > 0$) and the innovation is insignificant ($0 < \varepsilon < 2\delta$).*

Intuitively, on one hand, the intensive price competition after licensing drives down the industry profit when compared to the case of no licensing. On the other hand, when the innovation is nondrastic, the lower production cost of firm B compared to the no licensing case decreases the industry cost of production while some output produced by firm A before licensing is now produced by firm B after licensing, which may decrease or increase the industry cost of production depending on whether the patentee is inefficient or efficient (the change of the total industry cost of production is equal to $\left((c_A-\varepsilon)Q_A^F + (c_B-\varepsilon)\left(1-Q_A^F\right)\right) - \left((c_A-\varepsilon)Q_A^{NL} + c_B\left(1-Q_A^{NL}\right)\right) = -\delta\left(Q_A - Q_A^F\right) - \varepsilon\left(1-Q_A^{NL}\right)$): when the patentee is inefficient or equally efficient ($\delta \geq 0$), the total industry cost of production surely decreases after licensing; when the patentee is efficient, the total industry cost of production may increase or decreases after licensing depending on the degree of the patentee's inefficiency and the degree of the innovation. When the innovation is drastic, whether the total industry cost of production increases or decreases after licensing depends on whether the patentee is efficient or inefficient. It turns out that only when $\delta > 0$ and $0 < \varepsilon < 2\delta$, the industry profit increases and fixed fee licensing is better than no licensing for the patentee. In all other cases, the reverse is true.

2.4 Royalty Licensing

In the royalty regime, the cost-reducing innovation is sold to firm B using the royalty scheme. The maximum per-unit royalty that firm A can charge is ε. Let r denote the per-unit royalty. Then the two firms' profits can be expressed as $\pi_A = (p_A - (c_A-\varepsilon))\left(\frac{1}{2} + \frac{p_B-p_A}{t}\right) + r\left(\frac{1}{2} + \frac{p_A-p_B}{t}\right) = (p_A - (c_A - \varepsilon + r))\left(\frac{1}{2} + \frac{p_B-p_A}{t}\right) +$

[3] Since $\varepsilon > \delta + \frac{3}{2}t$ when the innovation is drastic, $9t\delta - 9t\varepsilon + 2\delta^2 + 9t^2 < 9t\delta - 9t\left(\delta + \frac{3}{2}t\right) + 2\delta^2 + 9t^2 = -\frac{1}{2}(3t-2\delta)(3t+2\delta) < 0$ (note $-\frac{3t}{2} < \delta < \frac{3t}{2}$).

r and $\pi_B = (p_B - (c_B - \varepsilon + r))\left(\frac{1}{2} + \frac{p_A - p_B}{t}\right)$. The equilibrium prices, demands and profits are given by the following:

$$p_A^r = c_A - \varepsilon + r + \frac{1}{6}(3t - 2\delta), \quad (19)$$

$$p_B^r = c_B - \varepsilon + r + \frac{1}{6}(3t + 2\delta), \quad (20)$$

$$Q_A^r = \frac{1}{6t}(3t - 2\delta), \quad (21)$$

$$Q_B^r = \frac{1}{6t}(3t + 2\delta), \quad (22)$$

$$\pi_A^r = \frac{1}{36t}(3t - 2\delta)^2 + r, \quad (23)$$

$$\pi_B^r = \frac{1}{36t}(3t + 2\delta)^2, \quad (24)$$

where the superscript r denotes per-unit royalty licensing case.

To maximize its profit, the patentee will choose $r = \varepsilon$ in stage 1. Thus, $\pi_A^r = \frac{1}{36t}(3t - 2\delta)^2 + \varepsilon$.

Now we compare firm A's profit in the royalty licensing case and in the no licensing case. When the innovation is nondrastic, $\pi_A^r - \pi_A^{NL} = \frac{1}{36t}(3t - 2\delta)^2 + \varepsilon - \frac{1}{36t}(3t + 2(\varepsilon - \delta))^2 = \frac{1}{9t}\varepsilon(6t + 2\delta - \varepsilon) > 0$; When the innovation is drastic, $\pi_A^r - \pi_A^{NL} = \frac{1}{36t}(3t - 2\delta)^2 + \varepsilon - \left(\varepsilon - \delta - \frac{t}{2}\right) = \frac{1}{36t}(9t + 2\delta)(3t + 2\delta) > 0$.

Also compare firm A's profit in the royalty licensing case and in the fixed fee licensing case. When the innovation is nondrastic, $\pi_A^r - \pi_A^F = \frac{1}{36t}(3t - 2\delta)^2 + \varepsilon - \frac{1}{36t}\left((3t - 2\delta)^2 + 4\varepsilon(3t + 2\delta - \varepsilon)\right) = \frac{1}{9t}\varepsilon(6t - 2\delta + \varepsilon) > 0$; When the innovation is drastic, $\pi_A^r - \pi_A^F = \frac{1}{36t}(3t - 2\delta)^2 + \varepsilon - \frac{1}{18t}\left(4\delta^2 + 9t^2\right) = \varepsilon - \frac{1}{36t}(3t + 2\delta)^2 > \delta + \frac{3t}{2} - \frac{1}{36t}(3t + 2\delta)^2 = \frac{1}{36t}(3t + 2\delta)(15t - 2\delta) > 0$.[4]

We thus have the following result.

Lemma 2 *In Salop's circular city model, royalty licensing is better than both no licensing and fixed licensing for the patentee.*

Royalty licensing is better than fixed licensing since the cost of production in these two cases are exactly the same while the price competition is less intense in the case of royalty licensing. Since firm B has the same profit in these two cases, firm A must be better off in the case of royalty licensing. It turns out that even though no licensing is better than fixed licensing when $\varepsilon > 2\delta$, royalty licensing is better than no licensing.

2.5 Two-part Tariff Licensing

This case is similar to royalty licensing except that there is a fixed fee. So $\pi_B^{TPT} = \frac{t}{4} - F^{TPT}$ and $\pi_A^{TPT} = \frac{t}{4} + r^{TPT} + F^{TPT}$, where the superscript TPT

[4] In fact, given Lemma 1 and the result we show above that royalty licensing is better than no licensing, we do not need to compare firm A's profit in the royalty licensing case and in the fixed fee licensing case when the innovation is drastic.

denotes two-part tariff licensing case. The patentee will choose $r^{TPT} = \varepsilon$ and $F^{^{TPT}}$ such that $\pi_B^{TPT} = \pi_B^r - F^{^{TPT}} = \pi_B^{NL}$. Thus, $F^{^{TPT}} = \pi_B^r - \pi_B^{NL}$. When the innovation is nondrastic, $F^{^{TPT}} = \frac{1}{36t}(3t + 2\delta)^2 - \frac{1}{36t}(3t - 2(\varepsilon - \delta))^2 = \frac{1}{9t}\varepsilon(3t + 2\delta - \varepsilon) > 0$; when the innovation is drastic, $F^{^{TPT}} = \frac{1}{36t}(3t + 2\delta)^2 > 0$. Therefore, we conclude:

Proposition 1 *In Salop circular' city model, the patentee's optimal licensing strategy is to license its innovation using two-part tariff no matter whether the patentee is ex ante efficient or inefficient or equally efficient as the other firm..*

Actually this must be better than royalty licensing since royalty licensing is just a special case of two-part tariff ($F^{^{TPT}} = 0$). Furthermore, as we have seen before, in the case of royalty licensing, since the market is fully covered by assumption, it is like each firm's marginal cost has been reduced by the same amount $\varepsilon - r$ but the patentee gets extra profit r, and due to less intensive price competition, firm B earns higher profit than in the case of no licensing. In two-part tariff, the patentee can extract the extra profit earned by firm B by setting a positive fixed fee.

3 Hotelling's model

Let us assume the patentee and the potential licensee are located at the end points of a linear city of unit length. Consumers are uniformly distributed over the linear city. Each buys exactly one unit of the good either from firm A or B. The transportation cost per unit of distance is t. The utility function of a consumer located at x is given by:

$$\begin{aligned} U &= v - p_A - tx && \text{if buys from firm A,} \\ &= v - p_B - t(1-x) && \text{if buys from firm B.} \end{aligned}$$

Assume the market is fully covered. It is straightforward to derive the demand for firms A and B, which is given below:

$$\begin{aligned} Q_A &= \frac{1}{2} + \frac{p_B - p_A}{2t} && \text{if } p_B - p_A \in [-t, t], \\ &= 0 && \text{if } p_B - p_A < -t, \\ &= 1 && \text{if } p_B - p_A > t, \end{aligned}$$

and

$$Q_B = 1 - Q_A.$$

3.1 Pre-innovation

Denote the marginal costs of production of A and B by c_A and c_B respectively and define $\delta = c_A - c_B$. We need to assume $-3t < \delta < 3t$ so that the less efficient firm's equilibrium quantity is positive before the innovation takes place. The equilibrium prices, demands and profits are given by the following:

$$p_A = \frac{1}{3}(3t + 2c_A + c_B) = c_A + \frac{1}{3}(3t - \delta), \quad (25)$$

$$p_B = \frac{1}{3}(3t + c_A + 2c_B) = c_B + \frac{1}{3}(3t + \delta), \quad (26)$$

$$Q_A = \frac{1}{6t}(3t - c_A + c_B) = \frac{1}{6t}(3t - \delta), \quad (27)$$

$$Q_B = \frac{1}{6t}(3t + c_A - c_B) = \frac{1}{6t}(3t + \delta), \quad (28)$$

$$\pi_A = \frac{1}{18t}(3t - c_A + c_B)^2 = \frac{1}{18t}(3t - \delta)^2, \quad (29)$$

$$\pi_B = \frac{1}{18t}(3t + c_A - c_B)^2 = \frac{1}{18t}(3t + \delta)^2. \quad (30)$$

As before firm A is the innovative firm (also called patentee) and it comes up with a cost reducing innovation which reduces the marginal cost of production by $\varepsilon \in (0, \min\{c_A, c_B\}]$. Firm B is the potential licensee. A licensing game consists of three stages as before.

Below, we list the equilibrium outcomes under all the relevant scenarios that we need to consider for our analysis.

3.2 No licensing

When $\varepsilon < 3t + \delta$, i.e., when the innovation is non-drastic, the equilibrium prices, demands and profits are given by the following:

$$p_A^{NL} = \frac{1}{3}(3t + 2(c_A - \varepsilon) + c_B) = c_A - \varepsilon + \frac{1}{3}(3t - \delta + \varepsilon), \quad (31)$$

$$p_B^{NL} = \frac{1}{3}(3t + (c_A - \varepsilon) + 2c_B) = c_B + \frac{1}{3}(3t + \delta - \varepsilon), \quad (32)$$

$$Q_A^{NL} = \frac{1}{6t}(3t - (c_A - \varepsilon) + c_B) = \frac{1}{6t}(3t - \delta + \varepsilon), \quad (33)$$

$$Q_B^{NL} = \frac{1}{6t}(3t + (c_A - \varepsilon) - c_B) = \frac{1}{6t}(3t + \delta - \varepsilon), \quad (34)$$

$$\pi_A^{NL} = \frac{1}{18t}(3t - (c_A - \varepsilon) + c_B)^2 = \frac{1}{18t}(3t - \delta + \varepsilon)^2, \quad (35)$$

$$\pi_B^{NL} = \frac{1}{18t}(3t + (c_A - \varepsilon) - c_B)^2 = \frac{1}{18t}(3t + \delta - \varepsilon)^2. \quad (36)$$

When the innovation is drastic, i.e., when $\varepsilon \geq \delta + 3t$, $p_A^{NL} = c_B - t$, $\pi_A^{NL} = c_B - t - (c_A - \varepsilon) = \varepsilon - \delta - t$, $\pi_B^{NL} = 0$.

3.3 Fixed Fee Licensing

In the fixed fee licensing case, the equilibrium prices, demands and profits are given by the following:

$$p_A^F = c_A - \varepsilon + \frac{1}{3}(3t - \delta), \tag{37}$$

$$p_B^F = c_B - \varepsilon + \frac{1}{3}(3t + \delta), \tag{38}$$

$$Q_A^F = \frac{1}{6t}(3t - \delta), \tag{39}$$

$$Q_B^F = \frac{1}{6t}(3t + \delta), \tag{40}$$

$$\pi_A^F = \frac{1}{18t}(3t - \delta)^2 + F, \tag{41}$$

$$\pi_B^F = \frac{1}{18t}(3t + \delta)^2 - F. \tag{42}$$

So the patentee will set the fixed fee equal to $\frac{1}{18t}(3t+\delta)^2 - \pi_B^{NL}$.

When the innovation is nondrastic, $F = \frac{1}{18t}(3t+\delta)^2 - \frac{1}{18t}(3t+\delta-\varepsilon)^2 = \frac{1}{18t}\varepsilon(6t+2\delta-\varepsilon) > 0$ since $0 < \varepsilon < \delta + 3t$. Firm A's profit is then $\pi_A^F = \frac{1}{18t}(3t-\delta)^2 + F = \frac{1}{18t}(3t-\delta)^2 + \frac{1}{18t}\varepsilon(6t+2\delta-\varepsilon) = \frac{1}{18t}\left((3t-\delta)^2 + \varepsilon(6t+2\delta-\varepsilon)\right)$. When the innovation is drastic, $F = \frac{1}{18t}(3t+\delta)^2$. Firm A's profit is $\pi_A^F = \frac{1}{18t}(3t-\delta)^2 + \frac{1}{18t}(3t+\delta)^2 = \frac{1}{9t}\left(\delta^2 + 9t^2\right)$.

Now we compare firm A's profit in the fixed fee licensing case and in the no licensing case. When the innovation is nondrastic, $\pi_A^{NL} - \pi_A^F = \frac{1}{18t}(3t-\delta+\varepsilon)^2 - \frac{1}{18t}\left((3t-\delta)^2 + \varepsilon(6t+2\delta-\varepsilon)\right) = \frac{1}{9t}\varepsilon(\varepsilon - 2\delta)$; When the innovation is drastic, $\pi_A^{NL} - \pi_A^F = \varepsilon - \delta - t - \frac{1}{9t}\left(\delta^2 + 9t^2\right) = -\frac{1}{9t}\left(9t\delta - 9t\varepsilon + \delta^2 + 18t^2\right) > 0.$[5] Hence, we have the following result.

Lemma 3 *In Hotelling's linear city model, no licensing is always better than fixed fee licensing for the patentee except that when the patentee is inefficient ($\delta > 0$) and the innovation is insignificant ($0 < \varepsilon < 2\delta$).*

As in the Salop model, whether no licensing is better or worse than fixed fee licensing depends on how the industry profit changes, which in turn depends on the intensity of price competition and the industry cost of production.

[5] Since $\varepsilon > \delta + 3t$ when the innovation is drastic, $9t\delta - 9t\varepsilon + \delta^2 + 18t^2 < 9t\delta - 9t(\delta + 3t) + \delta^2 + 18t^2 = (-3t+\delta)(3t+\delta) < 0$ (note $-3t < \delta < 3t$).

3.4 Royalty Licensing

In the royalty licensing case, the equilibrium prices, demands and profits are given by the following:

$$p_A^r = c_A - \varepsilon + r + \frac{1}{3}(3t - \delta), \tag{43}$$

$$p_B^r = c_B - \varepsilon + r + \frac{1}{3}(3t + \delta), \tag{44}$$

$$Q_A^r = \frac{1}{6t}(3t - \delta), \tag{45}$$

$$Q_B^r = \frac{1}{6t}(3t + \delta), \tag{46}$$

$$\pi_A^r = \frac{1}{18t}(3t - \delta)^2 + r, \tag{47}$$

$$\pi_B^r = \frac{1}{18t}(3t + \delta)^2. \tag{48}$$

To maximize its profit, the patentee will choose $r = \varepsilon$ in stage 1. Thus, $\pi_A^r = \frac{1}{18t}(3t - \delta)^2 + \varepsilon$.

Now we compare firm A's profit in the royalty licensing case and in the no licensing case. When the innovation is nondrastic, $\pi_A^r - \pi_A^{NL} = \frac{1}{18t}(3t - \delta)^2 + \varepsilon - \frac{1}{18t}(3t - \delta + \varepsilon)^2 = \frac{1}{18t}\varepsilon(12t + 2\delta - \varepsilon) > 0$; When the innovation is drastic, $\pi_A^r - \pi_A^{NL} = \frac{1}{18t}(3t - \delta)^2 + \varepsilon - (\varepsilon - \delta - t) = \frac{1}{18t}(9t + \delta)(3t + \delta) > 0$.

Also compare firm A's profit in the royalty licensing case and in the fixed fee licensing case. When the innovation is nondrastic, $\pi_A^r - \pi_A^F = \frac{1}{18t}(3t - \delta)^2 + \varepsilon - \frac{1}{18t}\left((3t - \delta)^2 + \varepsilon(6t + 2\delta - \varepsilon)\right) = \frac{1}{18t}\varepsilon(12t - 2\delta + \varepsilon)$; When the innovation is drastic, $\pi_A^r - \pi_A^F = \frac{1}{18t}(3t - \delta)^2 + \varepsilon - \frac{1}{9t}\left(\delta^2 + 9t^2\right) = \varepsilon - \frac{1}{18t}(3t + \delta)^2 > \delta + 3t - \frac{1}{18t}(3t + \delta)^2 = \frac{1}{18t}(15t - \delta)(3t + \delta) > 0$.

We thus have the following result.

Lemma 4 *In Hotelling's linear city model, royalty licensing is better than both no licensing and fixed licensing for the patentee.*

3.5 Two-part Tariff Licensing

We can show that the patentee will choose $r^{TPT} = \varepsilon$ and $F^{TPT} = \pi_B^r - \pi_B^{NL}$. When the innovation is nondrastic, $F^{TPT} = \frac{1}{18t}(3t + \delta)^2 - \frac{1}{18t}(3t + \delta - \varepsilon)^2 = \frac{1}{18t}\varepsilon(6t + 2\delta - \varepsilon) > 0$; when the innovation is drastic, $F^{TPT} = \frac{1}{18t}(3t + \delta)^2 > 0$. Therefore, we conclude:

Proposition 2 *In Hotelling's linear city model, the patentee's optimal licensing strategy is to license its innovation using two-part tariff no matter whether the patentee is ex ante efficient or inefficient or equally efficient as the other firm.*

4 Conclusion

We show that under spatial competitions when firms compete in prices, the optimal licensing strategy of an insider patentee is always two-part tariff irrespective of the size of innovation. This study gives us a simple justification of the prevalence of two-part tariff licensing as is reported in various empirical findings. This study also encompasses another realistic fact of competition among the firms, that is, in real life no two firms are actually symmetric and that is usually reflected in their cost of productions. Our study precisely captures that aspect and does a complete analysis based on all possible pre and post-innovation cost asymmetries between the patentee and licensee. We get a very robust theoretical result of optimal insider patent licensing, namely the two-part tariff as the dominant mode of licensing in all circumstances.

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