# 2016 AUT Mathematical Sciences Symposium

## Auckland University of Technology
## Auckland, New Zealand

1st – 2nd December 2016

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# Contents

## Welcome to the 2016 AUT Mathematical Sciences Symposium

On behalf of the Mathematical Sciences Research Group within the School of Engineering, Computer and Mathematical Sciences at Auckland University of Technology, we have much pleasure in welcoming you to the 2016 AUT Mathematical Sciences Symposium.

This is the third such Symposium and it is a continuation of our efforts to develop and promote the research being undertaken within the newly formed Department of Mathematical Sciences as part of our recently enlarged School. We are delighted to welcome a number of invited speakers to the Symposium with the aim of exploring collaborative opportunities and potential new areas of research that can be established with our research active staff.

The concept of this Symposium was a joint effort of ourselves. We both appreciate the assistance of Dr Kate Lee, and Dr Wenjun Zhang, who have each been involved in a variety of activities to ensure the continued success of this series.

As New Zealand's newest university we have recently had the opportunity to employ a number of new academic staff, all of whom have been developing research profiles. The School is putting in place a number of opportunities that will support and assist them in extending and enhancing their activities, with this meeting being one such effort.

Our growing postgraduate programme in the Mathematical Sciences at Honours, Masters and Doctoral levels has been enhanced with our Master of Analytics (MAnalytics) degree, now in its second year. The success of this programme, with twenty two students this year is leading to increased project supervision demands on our staff as well as leading to growing links with business and industry. We were delighted to sign a Partnering Agreement with Sovereign earlier in the year. We also have an arrangement with SAS Institute that will see students in our MAnalytics degree gaining SAS Certification on graduation.

We also look forward to the arrival in the new year of Professor Irene Hudson as she takes up the Chair in Statistics and Analytics. Irene will be a very welcome addition to our Analytics team and we cant wait for her arrival! We are also currently interviewing for an additional appointment in this area in order to strengthen our statistics staffing.

The Mathematical Sciences Research Group focuses on two main areas – Analytics and Applied Mathematics. We are very much focused on "research lead teaching" and we have developed a small number of research clusters within these areas to strengthen and support those academic staff working in these areas. Ideally we would like to foster collaborative activities and we thank those of you who have joined us at this meeting and we hope that we can facilitate some future joint research efforts.

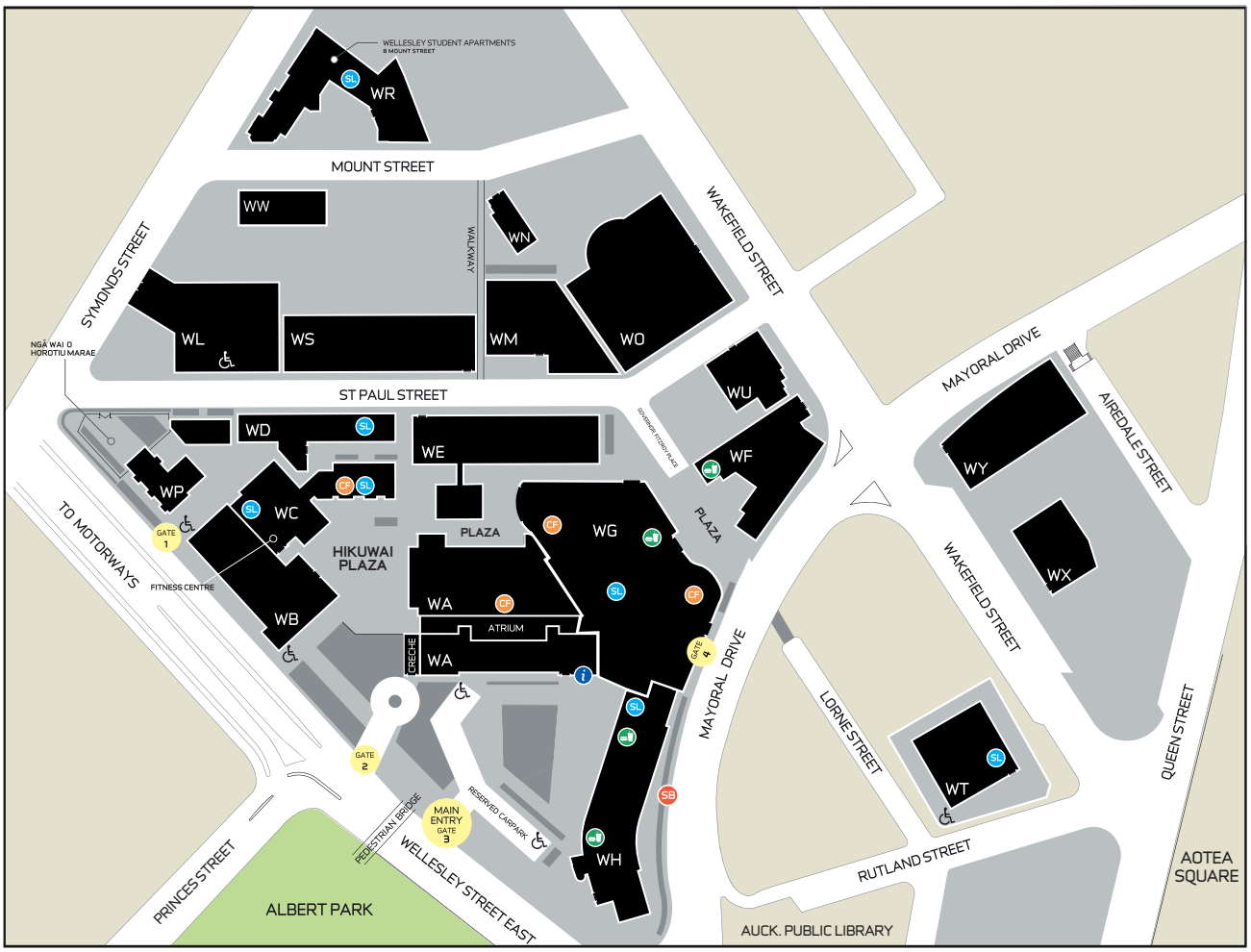
We have kept the focus narrow so as to make the meeting meaningful and rewarding for those who participate. We hope that you enjoy your time with us and that you find the exercise a useful adjunct to the mathematical and statistical scene within New Zealand.

*Jeffrey Hunter*
*Professor of Mathematical Sciences*
*Co-chair of the 2016 AUT Mathematical Sciences Symposium*

*Jiling Cao*
*Professor of Mathematics*
*Co-chair of the 2016 AUT Mathematical Sciences Symposium*

# AUT CITY CAMPUS

55 Wellesley Street East, Auckland 1010




## SCHOOLS

**Applied Sciences** – Level 5, WS building
**Art & Design** – Level 3, WE building
**Business** – Level 1, WF building
**Communication Studies** – Level 12, WG building
**Computer & Mathematical Sciences** – Level 1, WT building
**Engineering** – Level 3, WS building
**Hospitality & Tourism** – Level 3, WH building
**Language & Culture** – Level 8, WT building
**Law** – Level 6, WY building
**Social Sciences** – Level 14, WT building
**Te Ara Poutama** – Level 3, WB building

## AUT STUDENT CENTRE

Level 2, WA building

Phone: 0800 AUT UNI (0800 288 864)
Web: www.aut.ac.nz

## SERVICES AND FACILITIES

**AUT Student Centre** – Level 2, WA building
**Student Services Reception** – Level 1, WB building
**Library** – Level 4, WA building
**Early Childhood Centre** – Level 2, WA building via Gate 2
**AUT International Centre** – Ground Floor, WY building
**AuSM** – Level 2, WC building
**Safety & Security** – Corner St Paul & Wakefield St, WO building
**Health, Counselling & Wellbeing** – WB219, Level 2, WB building
**Printsprint customer service branch** – Level 3, WA building
**University Bookshop** – WC122, WC building

## JOIN A CAMPUS TOUR

Fridays at 2pm
AUT Student Centre

- AUT Student Centre
- Student lounge
- Cafés
- Conference facility
- Intercampus shuttle bus stop
- Mobility parks

# Symposium Schedule

| | Thursday 1st of December | | | Friday 2nd of December | |
|---|---|---|---|---|---|
| 8:45-9:00 | Registration (outside of WF710) | | | | |
| 9:00-9:10 | Welcome | | | | |
| 9:15-10:00 | *Plenary talk* Geoff McLachlan (WF710) | | 9:00-9:45 | *Plenary talk* Tak Kuen Siu (WF710) | |
| 10:05-10:25 | Roy Costilla (WF710) | Addison Pan (WF711) | 9:50-10:10 | Jiling Cao (WF710) | Murray Jorgensen (WF711) |
| 10:30-11:00 | Morning Tea | | 10:15-10:35 | Wenjun Zhang (WF710) | John Maindonald (WF711) |
| 11:00-11:45 | *Plenary talk* Graeme Wake (WF710) | | 10:35-11:10 | Morning Tea | |
| 11:50-12:10 | Jose Da Fonseca (WF710) | Richard Arnold (WF711) | 11:10-11:30 | Matthew Ryan (WF710) | Catherine Hassell Sweatman (WF711) |
| 12:15-12:35 | Yahua Xu (WF710) | Lynette Hunt (WF711) | 11:35-12:00 | Nina Anchugina (WF710) | Alna van der Merwe (WF711) |
| 12:35-14:00 | Lunch | | 12:05-12:50 | *Plenary talk* Rhema Vaithianathan (WF710) | |
| 14:00-14:45 | *Plenary talk* Mark McGuinness (WF710) | | 12:50-13:00 | Farewell | |
| 14:50-15:10 | Stefanka Chukova (WF710) | John Butcher (WF711) | | | |
| 15:15-15:35 | Robin Willink (WF710) | Alla Shymanska (WF711) | | | |
| 15:35-16:05 | Afternoon Tea | | | | |
| 16:05-16:25 | Robin Hankin (WF710) | Hyuck Chung (WF711) | | | |
| 16:30-17:15 | *Plenary talk* Jeffrey Hunter (WF710) | | | | |
| | | | | | |
| 18:30-21:00 | Symposium Dinner (Four Seasons Restaurant, AUT City Campus) | | | | |

# Symposium Information

### Location

The AUT Mathematical Sciences Symposium will be held in **WF Building**, corner of Wakefield Street and Mayoral Drive, Auckland Central.

### Registration

Registration will take place outside WF710.

### Presentations

Invited talks will be 40 minutes with 5 minutes for questions and contributed talks will be 20 minutes. There will be 5 minutes between 2 contributed talks for questions and to allow delegates to move between rooms

### Refreshments

Morning tea and afternoon tea will be served outside WF710. There are a large number of choices for lunch within a short walking distance of the campus.

### Dinner

The symposium dinner will begin at 6:30pm on Thursday 1st December 2016. The venue for the dinner is Four Seasons, WH Building, corner of Mayoral Drive and Wellesley Street East, Auckland.

The cost of dinner will be covered by Mathematical Sciences Research Group (MSRG) for participants who are presenting at the symposium. Partners are welcome to attend the dinner, however unfortunately the cost of their meals will not be covered by MSRG. If you need to pay for your or your partner's dinner, please discuss this with a member of the organising committee when you register.

### Further Queries

If you have any queries please do not hesitate to contact a member of the organising committee: Jiling Cao, Jeffrey Hunter, Kate Lee, Sarah Marshall and Wenjun Zhang.

# List of Authors and Titles

## Aggregating time preferences with decreasing impatience

Nina Anchugina, Matthew Ryan, Arkadii Slinko
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It is well-known that for a group of time-consistent decision makers their collective time preferences may become time-inconsistent. Jackson and Yariv (2014) demonstrated that the result of aggregation of exponential discount functions always exhibits strong present bias. As we show, when preferences satisfy the axioms of Fishburn and Rubinstein (1982), strong present bias is equivalent to strictly decreasing impatience (DI). Exponential discounting exhibits constant impatience. A related result is that of Weitzman (1988). He shows that under exponential discounting with an uncertain rate of time preference, long-term costs and benefits should be discounted at the lowest (i.e., most patient) possible rate of time preference. Weitzman's result has been influential in discussions of climate change. Applying the notion of comparative DI introduced by Prelec (2004), we generalize the result of Jackson and Yariv (2014). We prove that the aggregation of distinct discount functions from comparable DI classes results in the collective discount function which is strictly more DI than the least DI of the functions being aggregated. We also prove an analogue of Weitzman's (1998) result - our analogous result applies to hyperbolic rather than exponential discount functions. We show that if a decision maker is uncertain about her hyperbolic discount rate, then long-term costs and benefits will be discounted at a rate which is the probability-weighted harmonic mean of the possible hyperbolic discount rates.

## New statistical methods for tectonic stress estimation using earthquakes

Richard Arnold, Peter Jupp
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Tectonic stress estimation involves the interpretation of data that are intrinsically directional, or more commonly axial (e.g. earthquake focal mechanisms and shear wave splitting measurements). These observations are subject to observational error, noise (background as well as scattered seismic waves), axial ambiguities (e.g. strike directions) and physical ambiguities (fault normal vs slip direction). Robust inference from observational data requires properly formulated statistical models which use representations of oriented objects (vectors, planes, axes, frames), suitable statistics derived from these, and probability distributions describing their uncertainties. We discuss some recent developments in directional statistics that are relevant to the problem of stress estimation, as well as future research directions.

## Runge and his legacy

John Butcher
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The 1895 paper by Carl David Tolme' Runge, in which the first "Runge-Kutta method" was formulated, opened a completely new chapter in computational mathematics. An early triumph was the complete classification and analysis, by Martin Wilhelm Kutta (1901), of fourth order methods and the derivation of his famous method, based on Simpson's rule. Runge had a great admiration for the work of Kutta. In the early days of digital computers, Stanley Gill (1951) discovered a remarkable variation of the Kutta method which saved memory space at a time when this was a valuable commodity. The interest in Runge-Kutta methods has never subsided and new results, both theoretical and practical, have been discovered on a regular basis from the early days up to the present day. Some of the mathematical and computational advances, which form Runge's legacy, will be surveyed.

## Core versus Equilibria in Large Economies with Asymmetric Information

Jiling Cao
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Robert J. Aumann's (1964) core equivalence theorem states that the core in an atomless exchange economy is identical to the set of Walrasian equilibrium allocations. In 2000, E. Einy et al. showed that in an exchange economy with asymmetric information in which the space of traders is a measure space, and the set of states of nature is finite, if the economy is atomless and the utility function of each trader is measurable with respect to his information field, then the set of REE allocations is identical to the ex-post core. This is an extension of Robert J. Aumann's core equivalence theorem to economies with asymmetric information. In this talk, I will show that the result of E. Einy et al. can fail when an economy has infinitely many states of nature.

## Modelling Repairs of Systems with a Bathtub-Shaped Failure Rate Function

Stefanka Chukova, Sima Varnosafaderani, Richard Arnold
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Most of the reliability literature on modelling the effect of general repairs assumes that the system lifetime failure rate is monotonically increasing. These general repair models can be adapted to suit systems with bathtub-shaped failure rate (BFR) functions, but this requires the restrictive assumption of *only minimal repairs* during the initial, decreasing failure rate phase. In this paper, we propose a new approach to modelling the effect of general repairs on systems with a BFR function. The effect of a general repair is characterized as a modification in the conditional intensity function of the corresponding

failure process, such that the system following a general repair is at least as reliable as a system that has not failed.

## Acoustic wave scattering by an array of locally resonant cylinders

Hyuck Chung
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Acoustic wave transmission through a two-dimensional finite array of scatterers is investigated. Two types of scatterers are considered: sound-hard cylinders and C-shaped locally resonant scatterers. An analytical method is devised to solve the corresponding multiple scattering problems. The method combines an integral equation technique for the single scatterer with an enhanced multipole method using domain decomposition into slabs. Simulations of sound transmission through an array of 5 by 51 scatterers show remarkably good agreement with the corresponding in finite system. For an array comprising locally resonant scatterers, an approximate band gap around the resonator natural frequency is observed in addition to the band gap due to the overall periodicity of the sonic crystal array.

## To be or not to be (Bayesian) Non-Parametric: A tale about Stochastic Processes

Roy Costilla, Ivy Liu, Richard Arnold
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Thanks to the advances in the last decades in theory and computation, Bayesian Non-Parametric (BNP) models are now use in many fields including Biostatistics, Bioinformatics, Machine Learning, Linguistics and many others. Despite its name however, BNP models are actually massively parametric. A parametric model uses a function with finite dimensional parameter vector as prior. Bayesian inference then proceeds to approximate the posterior of these parameters given the observed data. In contrast to that, a BNP model is defined on an infinite dimensional probability space thanks to the use of a stochastic process as a prior. In other words, the prior for a BNP model is a space of functions with an infinite dimensional parameter vector. Therefore, instead of avoiding parametric forms, BNP inference uses an infinite number of them to gain more flexibility. To illustrate this, we present a simple but interesting example. We simulate count data from a Poisson mixture with three components and estimate it using a Dirichlet Process Mixture (DPM) prior. We show that this BNP model is tractable, i.e. is easily computed using Markov Chain Monte Carlo (MCMC) methods; allowing us to handle data with big sample sizes and estimate correctly the model parameters. Notably, these include the posterior distribution of the number of mixture components, an very important parameter in mixture modelling.

## Jump Activity Analysis for Affine Jump-diffusion Models: Evidences from the Commodity Market

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The objective of this paper is to perform a joint analysis of jump activity for commodities and their respective volatility indices. Exploiting the property that for affine jump-diffusion models a volatility index, which is quoted on the market, is an affine function of the instantaneous volatility state variable (thus turning this quantity observable), we perform a test of common jumps for multidimensional processes to assess whether an asset and its volatility jump together. Applying this test to the crude oil pair USO/OVX and the gold pair GLD/GVZ we find strong evidence that for these two markets the asset and its volatility have disjoint jumps. This result contrasts with existing results for the equity market and underpins a very specific nature of the commodity market. The results are further confirmed by analysing jump size distributions using a copula methodology.

## A liability allocation game

Robin Hankin
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The following problem is considered. Two players are each required to allocate a quota of n counters among $k$ boxes labelled $1,2,...k$. At times $t = 1,2,3,...$ a random box is identified; the probability of choosing box $i$ is $p_i$. If a player has at least one counter in the chosen box, she removes one counter from it; otherwise she takes no action. The winner is the first player to remove all her counters. This paper analyses this deceptively simple game, which has apparently not been studied in the literature.

## The trouble with sugar : modelling the contribution of hepatic glucose production to Type II diabetes

Catherine Hassell Sweatman
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It is generally accepted that consuming too much sugar on a daily basis may result in raised plasma glucose levels $G$ (pre-diabetes or Type II diabetes) and/or weight gain and, in some cases, $\beta$-cell failure (more severe Type II diabetes). However, hepatic glucose production (HGP) is also responsible for raising plasma glucose levels and the rate depends upon many factors including our insulin sensitivities. A mathematical model of glucose, insulin, glucagon, $\beta$-cell, leptin and fat dynamics and hepatic and peripheral insulin sensitivity is presented and the steady state behaviour is investigated. We study the relative contributions of excess dietary sugars and HGP under different assumptions concerning insulin sensitivity. The model predicts that a rapid rate of appearance of glucose in the blood followed by a

rapid transfer of this into fat is the least forgiving scenario, giving a shorter road ahead in parameter space (in extra dietary glucose intake), compromising our ability to survive a dietary imbalance.

## Comparing classical criteria for selecting intra-class correlated features for Three-mode Three-way data

Lynette Hunt, Kaye Basford
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Many unsupervised learning tasks involve data sets with both continuous and categorical attributes. One possible approach to clustering such data is to assume that the data to be clustered come from a finite mixture of populations. This approach requires the specification of the number of components to be fitted to the model and the form of the density functions of the underlying components. There has been extensive use of mixtures where the component distributions are multivariate normal and where the data would be described as two mode two way data. This talk investigates the behaviour of several commonly used model selection criteria when using the finite mixture model to cluster three way data containing mixed categorical and continuous attributes. We illustrate the performance of these criteria in selecting both the number of components in the model and the form of the correlation structure amongst the attributes when fitting a mixture model to several of the classical three way data sets.

## An academic career amidst applied probability and matrices - a journey with colleagues and mentors

Jeffrey Hunter
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The presenter has over his career made a number of contributions to a range of applied probability problems involving Markov chains, renewal processes, queueing processes, semi-Markov and Markov renewal processes. In some of these areas he has pioneered the use of generalized matrix inverses. Along this journey many individuals, colleagues, mentors and friends, have had an impact on his career. This presentation highlights some of those influences. A selection of some of the key results that appear in his published works in these fields of research are presented.

## Model Choice in Model-Based Clustering (MBC)

Murray Jorgensen
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I consider some of the choices that need to be made when implementing MBC such as the functional form of the components; within-component variable dependence; the addition of a uniform component. I will suggest that in the course of a study it may be important to re-fit the model several times as a more mechanistic understanding of the data starts to develop.

## Dose-Response Models that Accommodate Control Mortality

John Maindonald
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In the practical context that is in view here, insects other fruit or plant pathogens are exposed to varying "doses" of a fumigant. It has commonly been assumed that the probability p of mortality due to the treatment is independent of the control mortality, so that the total mortality $p^*$ can be expressed as $p^* = c + (1-c)p$. Under (unrealistic) binomial sampling assumptions, this would be a zero-inflated binomial model. Where a highly accurate estimates of the control mortality has been available, a model in which the complementary log-log transformation of p is a linear function of the dose has often worked well. The curvature introduced when the complementary log-log transformation of $p^*$ is then expressed as a function of dose can be accommodated using a low degree of freedom spline function. Estimates such as LD99 (dose that is expected to kill 99% of insects) are then straightforward to obtain. Confidence intervals are not. Fieller's formula requires adaptation to handle this inverse estimation problem. Suggestions?

## Goodness Gracious Great Balls of Fire

Mark McGuinness. Emma Greenbank, Ian Schipper, Andrew Fowler
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This is actually more about steam than about fire, although the context is that of a particular kind of volcanic eruption. I will describe modelling of the flashing of water to steam inside a hot erupted lava ball. A Surtseyan bomb is a lump of very hot magma that trails steam behind it when erupted from a volcano. These bombs are observed alongside cocks tails and cypress tree-like steam and magma emissions that are also features of volcanic eruptions where magma interacts with water on a large scale. Surtseyan ejecta are formed in shallow sub-aqueous eruptions. They occur when a combination of liquid water and sediments penetrates into bubbly molten magma during an eruption, and is then ejected from the volcano as an inclusion inside a porous ball of magma. After ejection there is a large temperature gradient between magma and inclusion. As the temperature of the inclusion increases, the liquid water vaporises causing a pressure increase inside the ejected ball, which can be modelled as a porous medium. The volcanological question is whether the ball of magma ruptures. Simple lumped calculations indicate the resulting steam pressures are expected to be well in excess of the tensile strength of rock. However, intact ejecta are easily found in Surtseyan debris, so we know that rupture does not always occur. Hence a more careful, transient, modelling approach is needed to explain and inform observations. We present partial differential equations that model transient changes in temperature and pressure in Surtseyan ejecta. We simplify the problem by approximating temperature behaviour to separate it from pressure. These equations are solved numerically and asymptotically to derive a parametric condition for rupture of ejecta. Steam escape times are also computed. We find that provided the permeability of the magma ball is large enough, steam escapes rapidly enough to relieve the high pressure developed at the flashing front, so that rupture does not occur. We describe a planned extension of this model that allows for the fact that pressure and temperature are in fact coupled - this is work in progress.

## Modelling and Clustering via Multivariate Skew Distributions

Geoff McLachlan
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Normal and t-mixture models have been widely applied to datasets from a variety of fields, including biostatistics, bioinformatics, economics, finance, and image analysis, among many others. Finite mixtures of multivariate skew-symmetric distributions, in particular, the skew normal and skew t-mixture models, are emerging as promising extensions to the traditional normal and t-mixture models. Most of these parametric families of skew distributions are closely related but they can vary widely in performance. In this talk, we give a brief overview of various existing proposals for multivariate skew distributions. Particular attention is given to the so-called canonical fundamental skew t-distribution. A number of illustrative examples are given to demonstrate how improved modelling and clustering results can be obtained via the latter model.

## A Generalization of Feddersen and Pesendorfer (1998): Voting Under Ambiguity

Addison Pan
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The well-known jury paradox - the more demanding the hurdle for conviction is, the more likely it is that a jury will convict an innocent defendant - heavily relies on Bayesian updating. However, with ambiguous information (e.g., a forensic test with accuracy of 60%, or more), standard Bayesian updating becomes invalid, challenging the existence of this paradox. By developing a novel theoretical model of voting under ambiguity, this study advances our understanding of how individuals process more realistically imprecise measures of information reliability and how this impacts on information aggregation for the group decision-making. Hence, our findings inform the institutional design of collective deliberation, for small to large group decision-making.

## Choice Behaviour that Exhibits 'Stochastic Betweenness': Characterisation and Testing

Matthew Ryan
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Choice behaviour appears to be inherently stochastic, and especially when choosing between "lotteries". A common mathematical structure for characterising (binary) stochastic choice is a Fechner model, in which choice probabilities are functions of utility differences. Refinements of Fechner models impose further restrictions on utility, A common refinement is linearity (i.e., the expected utility form). This refined model has mixed empirical performance. We explore a weaker restriction on utility than linearity

– one based on a 'betweenness' property. We establish necessary and sufficient conditions for choice behaviour to possess a Fechner representation of this form. We also consider the testable implications of this model.

## Mathematical and computational modeling of modulation-transfer function in electron-optical systems

Alla Shymanska
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The image quality and the resolving power of an optical device can be improved by improving its modulation-transfer-function (MTF). It is especially important for the devices which are designed for conditions of low illumination, such as a night vision. The case of infrared image converters and intensifiers with an inverting electron-optical system (EOS) and a micro-channel plate (MCP) as an amplifier is taken for consideration in this work. The effect of the electrostatic field penetration into the channel on the current-density distribution and the MTF of the system is under investigation in this work. The effect of the length of the contact conducting layer on the field penetration and MTF is evaluated. It is also shown how the emission from the conducting layer affects the resolution. The optimal parameters of EOS which provide a flat image surface to coincide with the channel plate are defined. Position of the best focus in the electron beam, what provides the highest resolution, is determined. Finally, the total MTF of the imaging device is calculated. The electron amplification process in the channel is simulated by Monte Carlo methods. Calculation of the electrostatic field in the device with rotational symmetry is a matter of finding a solution to the Laplace's partial differential equation in cylindrical coordinates. The system of equations of electron motion in the inhomogeneous electrostatic field is solved by the Runge-Kutta method in the cylindrical coordinate system.

## A Self-Exciting Threshold Jump-Diffusion Model for Option Valuation

Tak Kuen Siu
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In this talk, we shall discuss a self-exciting threshold jump-diffusion model for option valuation. This model incorporates regime switches without introducing an exogenous stochastic factor process. A generalized version of the Esscher transform is adopted to select a pricing kernel. The valuation of both the European and American contingent claims is considered. A piecewise linear partial differential-integral equation governing a price of a standard European contingent claim is obtained. For an American contingent claim, a formula decomposing a price of the American claim into the sum of its European counterpart and the early exercise premium is provided. An approximate solution to the early exercise premium based on the quadratic approximation technique is obtained for a particular case where the jump component is absent. Some numerical results for European and American options are presented for the case without jumps.

## Elastic waves in a locally linear Timoshenko beam

Alna van der Merwe
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The linear Timoshenko beam model consists of two partial differential equations formulated in terms of the deflection of the beam, and the angle due to rotation of a cross section. Shear and rotary inertia are taken into account in this model and in many practical applications this model compares well with higher dimensional beam models. A locally linear Timoshenko beam model has recently been derived from the laws of linear elasticity. Due to large displacements the system of partial differential equations are in general nonlinear for this model. Elastic waves in the beam are simulated using a mixed finite element method and the waves simulated by the linear and locally linear Timoshenko beam models are compared.

## Data science for social impact

Rhema Vaithianathan
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Social security and welfare was the largest area of expenditure by the New Zealand Government for the 2015/16 financial year. The success of these social interventions is impacted by the ability to target the individuals for whom they are most needed. The advent of data science and Big Data has brought considerable opportunity for agencies and institutions alike to better utilise existing data. Predictive risk models based on administrative data sets can identify individuals and communities at risk of negative outcomes. As a result, these individuals can be proactively targeted with preventive care and support services. This talk will evaluate the benefits of using predictive risk scoring tools to enhance front-line decision making, over the use of clinical judgment and threshold models. Drawing on recent examples, this talk will discuss the opportunities and challenges of predictive risk modelling in health and human service. This will include a discussion on the development and implementation of the first Child Welfare reactive predictive modelling tool, which is currently "live" in Allegheny County, US. A second predictive risk model will identify risk factors in California birth record data and discover communities where children have fared significantly better than expected. This talk will conclude with guidance on how the challenges to using predictive analytics are not technical, but rather how ethical and human output of predictive risk models can be turned into powerful interventions.

## Why the Black-Scholes formulation could not predict the *Black Monday* financial collapse in 1987

Graeme Wake, Heong-Hoon Kim
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Delta hedging is the core of the derivation of the well-known Black-Scholes formula for the price of European options. When Ito calculus is used faithfully without the self-financing hypothesis, the dependence of delta on both time and the underlying asset price variables occurs and a consistent version of the Black-Scholes equation is derived for the option price and the delta. This paper proposes the system as possible starting point of a more robust and correct study of mathematical option pricing.

## The test set should be 50 percent of the data

Robin Willink
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An analytics model should both perform well and be known to perform well. Its predictive ability can only be assessed honestly using data that is independent from the data on which the model was developed, so often a test set of data is set aside at the outset. But how large should this test set be? If it is too small then the model will not be optimal but if it is too small then the estimator of performance will have high variance. Sizes of 25 percent or 33 percent are typical, but it is difficult to find the question adequately formulated or the use of these figures justified. This talk considers choosing the size of the test set to make a lower confidence limit on model performance stochastically large. Theoretical results for trivial situations are supplemented by results with real datasets to conclude that 50 percent would be a more suitable figure.

## Higher Moment Risk Premiums for the Crude Oil Market

Yahua Xu
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Relying on options written on the USO, an exchange traded fund tracking the daily price changes of the WTI light sweet crude oil, we extract variance and skew risk premiums in a model-free way. We further decompose these risk premiums into downside and upside conditional components and show that they are time varying; that they can be partially explained by USO excess returns and, more importantly, these decomposed risk premiums enable a much better prediction of USO excess returns than the standard, or undecomposed, variance and skew risk premiums.

# Pricing variance swaps in a hybrid model of stochastic volatility and interest rate with regime-switching

Wenjun Zhang, Jiling Cao, Raihana Roslan

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This presentation considers the pricing of discretely-sampled variance swaps based on a hybrid model of stochastic volatility and stochastic interest rate with regime-switching. Our modeling framework extends the Heston stochastic volatility model by including the Cox-Ingersoll-Ross stochastic interest rate and parameters that switch according to a continuous-time observable Markov chain process. It is known that one limitation of the hybrid models is that the analytical pricing formula is often unavailable due to the non-affinity property of hybrid models. An efficient semi-closed form pricing formula is derived for the hybrid model. Our pricing formula is evaluated through numerical implementations to confirm its accuracy. Furthermore, the impact of including regime-switching on pricing variance swaps is also discussed.

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