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ABSTRACT

Current literature on public hospital efficiency in Australia only reveals information on how efficient public hospitals are in the short run. The presence of persistent technical inefficiency arising from long-term systemic problems and government-related regulatory constraints does not appear to have been addressed. Using yearly panel data for the period 2002-2018 on eight Australian states and territories, this study incorporates the measure of both transient and persistent technical inefficiency while controlling for unobserved heterogeneity to obtain a more precise measure of technical efficiency. The results of this study indicate that the technical inefficiency among public hospitals in Australia is persistent rather than transient based on state and territory level data. This implies that policymakers need to formulate comprehensive policies involving a longer time horizon that focuses on reducing the persistence in inefficiency among public hospitals in Australia. The study also calls on policymakers and regulators to disclose hospital-level data to researchers in order to gain further insight into the causes of persistence in inefficiency to formulate effective policies.

Keywords: stochastic frontier analysis; Australian hospitals; technical efficiency; persistent inefficiency; frontier estimation; Bayesian, STAN

1. Introduction

Like in many other developed countries, healthcare services in Australia account for a sizeable proportion of national, state and territory expenditures. Australia's healthcare expenditure is expected to increase in the coming decades to meet demand due to the ageing population, increasing incomes and consumer expectations (Australian Institute of Health Welfare, 2019; Australian Productivity Commission, 2015a). In 2017-2018, Australians spent nearly $185.4 billion (10% of GDP) on health products and services (i.e. government and non-governmental health spending), an average of $7,485 per capita (Australian Institute of Health Welfare, 2019).

The largest share of this budget (31.1%) was spent on public hospitals, with states and territories providing much of the funding, $29.9 billion (51.8%), followed by the Australian government spending of $22.7 billion (39.4%) and NGOs providing $5.1 billion (8.9%) (Australian Institute of Health Welfare, 2019). In the future, Australian government spending alone is expected to hit 5.7% of GDP by 2054-2055 (or $260 billion in current dollars), a 1.5% rise from 4.2% of GDP in 2014-2015 (Australian Productivity Commission, 2015a).

In light of increasing healthcare expenditure, understanding the efficiency of the health system, including public hospitals, has become an essential issue for government, healthcare funders and regulators (Australian Productivity Commission, 2015a; Wang *et al.*, 2006). Furthermore, efficiency measurement of public hospitals provides a measure to estimate how effectively public funds are utilised in providing hospital services.

The literature on the efficiency of Australian public hospitals is thin, with studies limited to hospitals in specific states and territories (Nghiem *et al.*, 2011; Wang *et al.*, 2006). An exception in the Australian literature is a study commissioned by the Australian Productivity Commission (Forbes *et al.* 2010) which undertook the technical efficiency analysis of selected private and public acute hospitals in Australia for the period 2004-2007. While their study estimated the transient level of technical efficiency, it overlooked the persistent technical inefficiency that lies hidden within the unobserved heterogeneity. According to Tsionas and Kumbhakar (2014) and Badunenko and Kumbhakar (2016), ignoring the persistent nature of technical inefficiency causes an upward bias in the measure of efficiency, which can lead to incorrect policy implications.

We approached the Australian Bureau of Statistics and the Australian Institute of Health Welfare to obtain more recent data similar to those used by Forbes *et al*. (2010). Our intention with this study was to compute transient and persistent technical inefficiency while simultaneously controlling for unobserved heterogeneity. However, our request for data was denied, possibly due to restricted access and the sensitive nature of the data. Therefore, we had to use aggregated public hospital data for each state and territory from annual health reports available on the Australian Institute of Health Welfare website. A report by the Australian Productivity Commission (2015b) highlighted the lack of access to healthcare data sets as a barrier in quantifying inefficiency in the Australian healthcare system.

In general, the measurement of persistent technical inefficiency is often ignored in healthcare efficiency analysis. We suspect this is because no readily available software currently enables researchers to run models that can integrate persistent inefficiency. As a result, such analysis is limited to practitioners with strong programming skills. Therefore, the primary purpose of this paper is to fill this gap in efficiency literature both in terms of study and practice.

In general, this study is novel in three ways:

- **i.** It undertakes first efficiency analysis that incorporates the persistent nature of technical efficiency while controlling for unobserved heterogeneity among Australian states and territories to obtain an unbiased estimate of technical efficiency;
- **ii.** It uses less utilised, but highly flexible, gamma distribution to model persistent inefficiency using a finite mixture stochastic frontier model;
- **iii.** The STAN code used in this study is made available so it can be replicated using other data sets in healthcare or in other sectors.

2. Methodology

While this study's estimation methodology is based on the well-developed stochastic frontier production model proposed by Meeusen and Van den Broeck (1977) and Aigner *et al*. (1977), this study also incorporates the finite mixture model proposed by Griffin and Steel (2008) to capture and separate persistent inefficiency from unobserved heterogeneity.

On the basis of the first application of Bayesian techniques to stochastic frontier models by van den Broeck *et al.* (1994) and Koop *et al.* (1995) the Markov Chain Monte Carlo (MCMC) simulation is used to derive the posterior moments of the model parameters.

The estimation of technical efficiency requires an estimation of production or distance function. A translog (Christensen *et al.*, 1973) input-distance function with multiple hospital outputs with an estimable form can be represented as:

$$\begin{aligned} -\ln x_{Mit} = \ & \alpha_0 + \sum_{n=1}^{N} \alpha_n \ln y_{nit} + \frac{1}{2} \sum_{n=1}^{N} \sum_{k=1}^{N} \alpha_{nk} \ln y_{nit} \ln y_{kit} \\ & + \sum_{m=1}^{M-1} \beta_m \ln x_{mit}^* + \frac{1}{2} \sum_{m=1}^{M-1} \sum_{l=1}^{M-1} \beta_{ml} \ln x_{mit}^* \ln x_{lit}^* \\ & + \sum_{m=1}^{M-1} \sum_{n=1}^{N} \delta_{mn} \ln x_{mit}^* \ln y_{nit} + \sum_{n=1}^{N} \omega_n \ln y_{nit}\, l + \sum_{m=1}^{M-1} \theta_m \ln x_{mit}^*\, l \\ & + \gamma_t l + \frac{1}{2} \vartheta_{tt} l^2 + v_{it} - u_{it} - \alpha_i \\ & t = 1, \dots, T \qquad i = 1, \dots, n. \end{aligned} \tag{1}$$

where T is the number of observations for state or territory i, whereas $\ln y_{nit}$ and $\ln x_{mit}$ represent the natural logarithm of quantities for outputs and inputs, respectively. The variable $x_{mit}^* = \frac{x_{mit}}{x_{Mit}}$, with x_{Mit} being the normalising input imposes the linear homogeneity in inputs. Furthermore, $l = 1, \dots, T$ is a linear time trend, whereas l^2 captures the quadratic trend in the input-distance function. The variable $v_{it} \sim N(0, \sigma_v$[1]$)$ captures the stochastic noise, whereas the inefficiency variable u_{it} is assumed to be gamma-distributed, i.e. $u_{it} \sim Ga(\phi_u, \lambda_u)$, which is also the measure of transient technical inefficiency. The measure of transient technical efficiency can be found by $exp(-u_{it})$.

The variable α_i in equation (1) is the mixture of time-invariant unobserved heterogeneity and persistence inefficiency of each state and territory which can be represented as:

$$\alpha_i = \rho_i(\alpha_i | 0, \sigma_\alpha^2) + 1 - \rho_i(\alpha_i | \phi_\alpha, \lambda_\alpha) \tag{2}$$

where the variable $\rho_i \in (0,1)$ is the mixing proportion, with $\rho_i \sim beta(\gamma_\rho, \tau_\rho)$. Further in equation (2), the first part in $\alpha_i \sim N(0, \sigma_\alpha)$ controls for the state- or territory-specific heterogeneity whereas the second part, $\alpha_i \sim Ga(\phi_\alpha, \lambda_\alpha)$ captures the persistent technical

[1] Where σ denotes standard deviation.

inefficiency for each state and territory. The measure of persistent technical efficiency can be estimated by $exp(-(1-\rho_i(\alpha_i|\phi_\alpha,\lambda_\alpha)))$.

As Bayesian techniques are used to estimate model parameters in equations (1) and (2), it is essential to specify the complete data likelihood of the structural parameters. For succinctness, equation (1) is rewritten as:

$$q_{it} = p'_{it}\boldsymbol{\beta} + v_{it} - u_{it} - \alpha_i, \qquad (3)$$

where q_{it} represents the dependent variable, p'_{it} is a row vector of the independent variables in equation (1) and $\boldsymbol{\beta}$ is the vector of corresponding parameters, including the intercept that is to be estimated. If all the parameters in equations (2) and (3) are collected into a vector $\theta = [\boldsymbol{\beta}, \sigma_v]'$, then the complete data likelihood of the structural parameters is:

$$p(\boldsymbol{q}, \{\alpha_i\}, \{\rho_i\}, \{u_{it}\} \,|\, \boldsymbol{\theta}, \boldsymbol{P}) = p(q \,|\, \{\alpha_i\}, \{u_{it}\}, \beta, \sigma_v, \boldsymbol{P}) \times p(\{\alpha_i\} \,|\, , \{\rho_i\}) \qquad (4)$$

$$= \left[\frac{1}{(2\pi\sigma_v^2)^{\frac{NT}{2}}}\, exp\left\{-\frac{\sum_{i=1}^{N}\sum_{t=1}^{T-1}(q_{it} - \alpha_i - \boldsymbol{p}'_{it}\beta - u_{it})^2}{2\sigma_u^2}\right\}\right]$$

$$\times \left[\rho_i\left(\frac{1}{(2\pi\sigma_\alpha^2)^{\frac{N}{2}}}\, exp\left\{-\frac{\sum_{i=1}^{N}\alpha_i^2}{2\sigma_\alpha^2}\right\}\right) + (1-\rho_i)\left(\frac{\lambda_\alpha^{\phi_\alpha}\sum_{i=1}^{N}\alpha_i^{\phi_\alpha-1}\, exp\{-\lambda_\alpha\sum_{i=1}^{N}\alpha_i\}}{\Gamma(\phi_\alpha)}\right)\right]$$

where $\boldsymbol{q}$ and $\boldsymbol{P}$ are the stacked vector and matrix, over both i and t as in equation (3) respectively. The symbol $\Gamma(.)$ denotes gamma function.

Using Bayes' rule, the posterior density of model parameters are

$$\pi\left(\theta, \{\alpha_i\}, \{u_{it}\}, \{\rho_i\} \,\middle|\, \boldsymbol{q}, \boldsymbol{P}\right) \propto p\left(\boldsymbol{q}, \{\alpha_i\}, \{\rho_i\}, \{u_{it}\} \,\middle|\, \boldsymbol{\theta}, \boldsymbol{P}\right) \times p(\theta) \qquad (4)$$

We impose weakly informative priors on the parameters as follows:

1. The priors of vectors $\boldsymbol{\beta}$ is a normal density with mean zero and with a standard deviation of five. Similarly, for the heterogeneity part of α_i the prior is $N(0, 5)$ whereas for the persistent inefficiency part of α_i and transient inefficiency u_{it} is assigned a gamma prior with shape and scale with normal hyper priors of zero and five is used.. For the σ_v a prior of $Ga(1,1)$ is used.
2. For the mixing parameter ρ a beta prior with shape parameters with normal hyper priors with mean zero and with a standard deviation of five is used.

We used state-of-the-art 'No-U-Turn' Sampler (NUTS[2]), which is a highly efficient extension of the Hamiltonian Markov Chain Monte Carlo (HMCMC) algorithm, to draw samples from the posterior distribution. The MCMC sampling process was undertaken using RStan[3], and the full STAN code used for modelling is provided in Appendix A3. The sampling process involves five independent Markov chains with each chain contributing 80,000. As NUTS is much more efficient in getting samples with lower autocorrelations compared to Gibbs sampling and Metropolis algorithm, no burn-ins or thinning of chains is required (Stan Development Team, 2020).

3. Data and descriptive statistics

Data for this study included balanced panel data for the period 2002-2018 on public hospitals for six Australian states[4] and two territories[5] on outputs, labour inputs, capital input and non-salaried costs. The outputs are proxied by the number of separations and outpatient services. The labour inputs include average full-time equivalents (FTEs) counts of medical officers, nurses, diagnostic and allied staff, administrative and clerical staff, and personal care staff. The FTEs of nurses, diagnostic and allied staff, administrative and clerical staff, and personal care staff are aggregated based on their respective proportion of the total labour expenditure. This is consistent with the classic assumption that a cost-minimising firm pays its staff according to their marginal products.

In terms of capital inputs, the commonly used proxy in the healthcare literature is the number of hospital beds (see, for example, studies by Aletras *et al*. (2007), Herr (2008), Cozad and Wichmann (2013), Asmild *et al.* (2013) and Mitropoulos *et al*. (2015)). Therefore, the average number of public hospital beds for each state and territory is used as a proxy for capital input. Further, to account for non-labour expenditure, a measure of all other clinical and day-to-day running costs are also included in the study. The full definition of all the variables and the corresponding summary statistics are displayed in Table 1.

[2] Hoffman and Gelman (2014).
[3] Stan Development Team (2018). *RStan: The R interface to Stan*. R package version 2.17.3. http://mc-stan.org
[4] Includes New South Wales, Queensland, South Australia, Tasmania, Victoria and Western Australia.
[5] Includes Australian Capital Territory and Northern Territory.

4. Empirical results and discussion

Before the estimation of the translog cost function, the data is normalised by its geometric mean to allow the interpretation of parameters associated with the first-order terms directly as distance elasticities. The homogeneity restriction on inputs in the translog input-distance function is imposed by dividing all the inputs by the medical officer FTE (med) counts. In Table 2, the posterior means, standard deviations and 95% credible interval are displayed for all the parameters of the estimated translog input-distance function. The trace plot of HMCMC associated with the first-order parameter and the standard deviation are displayed in Appendix A2. The trace plot shows that the chains have mixed well, which suggests that the sample generated from the HMCMC algorithm is sufficient to provide an accurate approximation of the target distribution.

The first-order parameters displayed in Table 2 show that the sign of input parameters have expected signs. The posterior mean of *separations* and *outpatients* add up to -0.31, which is also the scale elasticity of the input-distance function. The estimated scale elasticity implies that a 1% increase in all inputs results in approximately a 3.2 % increase in the number of separations and outpatients in state and territory hospitals. The first order coefficient of time (*trend*) is -0.02, which translates to the technological progress of 2% among Australian public hospitals.

In order to check how well the fitted model compares to the observed dependent variable, a posterior predictive check (Gelman & Hill, 2006) is undertaken where we simulate 10,000 samples from the posterior predictive distribution. The plot of the simulated samples and observed dependent variable is displayed in Appendix A3, which suggests that the fitted model compares well to the observed dependent variable.

In Table 3, the average transient and persistent technical efficiency scores for each state and territory are ranked according to their performance. The average transient technical efficiency stands at 0.95; however, persistent technical efficiency is significantly low, with an average of 0.50. As a result of higher persistence in technical inefficiency, the total[6] average technical efficiency is pulled down to an average of 0.47. Therefore, the inefficiency in the Australian

[6] Total technical efficiency score is obtained by the product of transient and persistent technical efficiency as shown in Kumbhakar *et al*. (2015).

public hospital system appears to be persistent rather than transient. According to Colombi *et al.* (2017), in the healthcare sector the existence of persistent technical inefficiency points towards long-run moral hazards such as substandard infrastructure, labour rigidity (lower standard of human capital) and inefficient internal organisation.

A report by the Australian Productivity Commission (2015b) highlighted the existence of substantive duplicative and irrelevant healthcare intervention along with restrictions on healthcare professionals. Various organisations including the Commonwealth, state and territory governments, and non-government sectors are responsible for service provision, funding, policy and regulation of health care in Australia. The fragmented and complexity of the Australian health system has resulted in duplication, waste, and gaps in service delivery, cost shifting among multiple parties and difficulty in reforming the sector (Australian Productivity Commission, 2015a). These are potentially the driving forces behind the high inefficiency observed in our study and increases in healthcare expenditure observed in Australia over the past few decades (Australian Institute of Health and Welfare, 2019).

Further, the state/territory level analysis of the efficiency scores in Table 3 shows that there is a substantial inefficiency gap between the top and the lowest performers. The top three states – Western Australia, Victoria and Tasmania – have technical efficiency that ranges between 0.77-0.60. However, the bottom five – Queensland, South Australia, Northern Territory, New South Wales and Australian Capital Territory – posted an average score between 0.29-0.38. This suggests that these bottom-performing states and territories have operated with a very high amount of persistence in technical inefficiency between 2002-2018. Reducing the level of persistence in inefficiency would lead to greater access to hospital care, better health outcomes and higher quality of care for a given level of public funding. It would be of immense assistance to policymakers and regulators to assess the persistent technical efficiency of individual hospitals in these lowest-performing states and territories.

5. Conclusion and policy implications

In conclusion, this research is the first in Australian healthcare literature to introduce the measurement of persistent technical inefficiency along with transient inefficiency to gain an understanding of the form of inefficiency that most requires policymakers' attention. The results highlight the fact that the inefficiency among Australian public hospitals is persistent rather than transient. This requires a thorough look at the existing funding models and/or regulations that may be driving persistence in inefficiencies among public hospitals. Although the reduction of persistent inefficiency is a long-term goal, the identification of persistent inefficiency and its source is key to the provision of sustainable public hospital services in Australia.

Finally, we hope that the findings of this paper will attract the attention of the Australian public authorities, who will then be more willing to share detailed data, especially at the provider level. This would then facilitate more rigorous research and thereby provide significantly more insight into efficiency performance and allow researchers to provide more detailed policy implications.

References

Aigner, D., Lovell, C.A.K. & Schmidt, P. 1977, 'Formulation and estimation of stochastic frontier production function models', *Journal of Econometrics*, vol. 6, no. 1, pp. 21-37.

Aletras, V., Kontodimopoulos, N., Zagouldoudis, A. & Niakas, D. 2007, 'The short-term effect on technical and scale efficiency of establishing regional health systems and general management in Greek NHS hospitals', *Health Policy*, vol. 83, no. 2, pp. 236-45.

Asmild, M., Hollingsworth, B. & Birch, S. 2013, 'The scale of hospital production in different settings: one size does not fit all', *Journal of Productivity Analysis,* vol. 40, no. 2, pp. 197-206.

Australian Bureau of Statistics 2020, *Producer Price Indexes, Australia*, https://www.abs.gov.au/statistics/economy/price-indexes-and-inflation/producer-price-indexes-australia/latest-release.

Australian Institute of Health and Welfare 2020, *Glossary*, https://www.aihw.gov.au/reports-data/myhospitals/content/glossary.

Australian Institute of Health Welfare 2019, *Health expenditure in Australia*, https://www.aihw.gov.au/getmedia/80dcaae7-e50f-4895-be1f-b475e578eb1b/aihw-hwe-77.pdf.aspx?inline=true.

Australian Productivity Commission 2015a, *Efficiency in Health*, https://www.pc.gov.au/research/completed/efficiency-health.

Australian Productivity Commission 2015b, *Improving Australia's health system: what we can do now*, https://www.pc.gov.au/news-media/pc-news/previous-editions/pc-news-may-2015/improving-australia-health-system.

Badunenko, O. & Kumbhakar, S.C. 2016, 'When, where and how to estimate persistent and transient efficiency in stochastic frontier panel data models', *European Journal of Operational Research*, vol. 255, no. 1, pp. 272-87.

Christensen, L.R., Jorgenson, D. & Lau, L.J. 1973, 'Transcendental Logarithmic Production Frontiers', *The Review of Economics and Statistics*, vol. 55, no. 1, pp. 28-45.

Colombi, R., Martini, G. & Vittadini, G. 2017, 'Determinants of transient and persistent hospital efficiency: The case of Italy', *Health Economics*, vol. 26, no. S2, pp. 5-22.

Cozad, M. & Wichmann, B. 2013, 'Efficiency of health care delivery systems: effects of health insurance coverage', *Applied Economics*, vol. 45, no. 29, pp. 4082-94.

Forbes, M., Harslett, P., Mastoris, I. & Risse, L. 2010, *Measuring the technical efficiency of public and private hospitals in Australia*.

Gelman, A. & Hill, J. 2006, 'Simulation for checking statistical procedures and model fits', in *Data Analysis Using Regression and Multilevel/Hierarchical Models*, Cambridge University Press, Cambridge, pp. 155-66.

Griffin, J.E. & Steel, M.F.J. 2008, 'Flexible mixture modelling of stochastic frontiers', *Journal of Productivity Analysis*, vol. 29, no. 1, pp. 33-50.

Herr, A. 2008, 'Cost and technical efficiency of German hospitals: does ownership matter?', *Health Economics*, vol. 17, no. 9, pp. 1057-71.

Hoffman, M.D. & Gelman, A. 2014, 'The No-U-Turn sampler: adaptively setting path lengths in Hamiltonian Monte Carlo', *J. Mach. Learn. Res.*, vol. 15, no. 1, pp. 1593-623.

Koop, G., Steel, M. & Osiewalski, J. 1995, 'Posterior Analysis of Stochastic Frontier Models Using Gibbs Sampling', *Computational Statistics*, vol. 10, no. 10, pp. 353-73.

Kumbhakar, S.C., Wang, H.-J. & Horncastle, A.P. 2015, *A Practitioner's Guide to Stochastic Frontier Analysis Using Stata*, Cambridge University Press, Cambridge.

Meeusen, W. & Van den Broeck, J. 1977, 'Efficiency Estimation from Cobb-Douglas Production Functions with Composed Error', *International Economic Review*, vol. 18, no. 2, pp. 435-44.

Mitropoulos, P., Talias, M.A. & Mitropoulos, I. 2015, 'Combining stochastic DEA with Bayesian analysis to obtain statistical properties of the efficiency scores: An application to Greek public hospitals', *European Journal of Operational Research*, vol. 243, no. 1, pp. 302-11.

Nghiem, S., Coelli, T. & Barber, S. 2011, 'Sources of Productivity Growth in Health Services: A Case Study of Queensland Public Hospitals', *Economic Analysis and Policy*, vol. 41, no. 1, pp. 37-48.

Stan Development Team 2020, *Markov Chain Monte Carlo*, https://bookdown.org/marklhc/notes_bookdown/markov-chain-monte-carlo.html.

Tsionas, E.G. & Kumbhakar, S.C. 2014, 'Firm heterogeneity, persistent and transient technical inefficiency: A generalised true random-effects model', *Journal of Applied Econometrics*, vol. 29, no. 1, pp. 110-32.

van den Broeck, J., Koop, G., Osiewalski, J. & Steel, M.F.J. 1994, 'Stochastic frontier models: A Bayesian perspective', *Journal of Econometrics*, vol. 61, no. 2, pp. 273-303.

Vehtari, A. & Ojanen, J. 2012, 'A survey of Bayesian predictive methods for model assessment, selection and comparison', *Statist. Surv.*, vol. 6, pp. 142-228.

Wang, J., Zhao, Z. & Mahmood, A. 2006, 'Relative efficiency, scale effect, and scope effect of public hospitals: evidence from Australia'.

TABLES

Table 1

Summary statistics of variables

Variable	Description†	Statistics	
Labour inputs (FTEs)			
Medical officers *(med)*	Refers to medical officers employed by the hospital on a full-time or part-time salaried basis. This excludes visiting medical officers engaged on an honorary, sessional or fee-for-service basis. This category includes salaried medical officers who are engaged in administrative duties.	Minimum	235.00
		Maximum	13,614.00
		Mean	3,799.41
		Standard deviation	3,586.67
Nurses & other staff *(nur_other_staff)*	This includes the weighted FTE measure of nurses, diagnostic and allied health professionals, administrative and clerical staff, and domestic and other personal care staff.	Minimum	1,077.84
		Maximum	98,749.76
		Mean	25,678.31
		Standard deviation	24,918.63
Capital and other inputs			
Average number of beds *(beds)*	Refers to the average number of beds immediately available for use (with staffing).	Minimum	358.00
		Maximum	21,253.00
		Mean	7,106.28
		Standard deviation	6,520.40

Non-salary expenditure[§] *(other_exp)*	Includes payments to visiting medical officers, superannuation, drug supplies, medical and surgical supplies, food supplies, domestic services, repairs and maintenance, patient transport, administrative expenses, interest, lease costs, other on-costs and other recurrent expenditure.	Minimum Maximum Mean Standard deviation	64.25 11,747.73 2,342.50 2,719.50
Outputs			
Case-mix adjusted separations *(separations)*	The total number of episodes of care (also hospitalisations) for admitted patients, which can be total hospital stays (from admission to discharge, transfer or death) or portions of hospital stays beginning or ending in a change of type of care. The separations are case mix adjusted by taking the product of total separations and average cost weight.	Minimum Maximum Mean Standard deviation	45,739.46 1,972,644.00 630,127.70 585,259.40
Outpatients occasions of service *(outpatients)*	A distinct visit to a hospital or outpatient clinic where treatment is received without being admitted. As a person may visit an outpatient clinic in a hospital more than once in a year, the number of occasions of service is not the same as the number of people treated in outpatient clinics	Minimum Maximum Mean Standard deviation	190,500.00 28,100,000.00 5,010,165.00 5,815,371.00

[†] Based on the definition provided in the Australian Institute of Health and Welfare (2020).

[§] Reported in millions of Australian Dollars and deflated by the national-level producer price index for Medical & surgical equipment manufacturing sector available at Australian Bureau of Statistics (2020).

Table 2

Posterior means and structural parameters

Variable	Posterior Mean	Standard Deviation	95% credible interval
log_separations	-0.26	0.09	[-0.42, -0.08]
log_outpatients	-0.05	0.02	[-0.09, -0.01]
log_nur_other_staff	0.42	0.07	[0.27, 0.57]
log_other_exp	0.07	0.03	[0.01, 0.12]
log_beds	0.63	0.06	[0.51,0.75]
log_nur_other_staff × *log_other_exp*	0.23	0.26	[-0.28,0.72]
log_nur_other_staff × *log_beds*	-0.82	0.53	[-1.87, 0.20]
log_nur_other_staff × *log_separations*	0.27	0.16	[-0.05, 0.59]
log_nur_other_staff × *log_outpatients*	-0.28	0.15	[-0.57, 0.00]
log_nur_other_staff ^{2}	-1.18	0.86	[-2.82, 0.53]
log_other_exp × *log_beds*	0.04	0.23	[-0.41, 0.49]
log_other_exp × *log_separations*	-0.17	0.07	[-0.32, 0.03]
log_other_exp × *log_outpatients*	0.08	0.06	[-0.04, 0.20]
log_other_exp ^{2}	-0.27	0.21	[-0.68, 0.14]
log_beds × *log_separations*	0.22	0.18	[-0.13, 0.57]
log_beds × *log_outpatients*	-0.08	0.17	[-0.42, 0.25]
log_beds ^{2}	2.52	0.33	[1.87,3.17]
log_separations × *log_outpatients*	0.04	0.05	[-0.07, 0.14]
log_separations ^{2}	-0.23	0.10	[-0.43, -0.03]
log_outpatients ^{2}	0.01	0.05	[-0.08, 0.11]
trend	0.02	0.00	[0.01, 0.03]
trend ^{2}	0.002	0.00	[0.00, 0.004]

trend × *log_separations*	-0.04	0.01	[-0.06, -0.01]
trend × *log_outpatients*	0.02	0.01	[0.00, 0.04]
trend × *log_nur_other_staff*	0.12	0.03	[0.07, 0.17]
trend × *log_other_exp*	-0.02	0.02	[-0.06, 0.02]
trend × *log_beds*	-0.16	0.03	[-0.21, -0.11]
constant	1.97	0.56	[0.67, 2.97]
Standard deviation			
$\boldsymbol{\sigma_v}$	0.02	8.9×10^{-3}	[0.003, 0.03]
log posterior density (Vehtari & Ojanen 2012)	50.33		

Table 3

Estimates of average technical efficiency scores

State/Territory	Technical Efficiency			Rank
	Transient	Persistent	Total	
Western Australia	0.95	0.82	0.77	1
Victoria	0.95	0.72	0.67	2
Tasmania	0.94	0.64	0.60	3
Queensland	0.95	0.40	0.38	4
South Australia	0.95	0.41	0.38	4
Northern Territory	0.94	0.34	0.32	5
New South Wales	0.95	0.33	0.31	6
Australian Capital Territory	0.94	0.30	0.29	7
Grand Mean	**0.95**	**0.50**	**0.47**	

Appendix

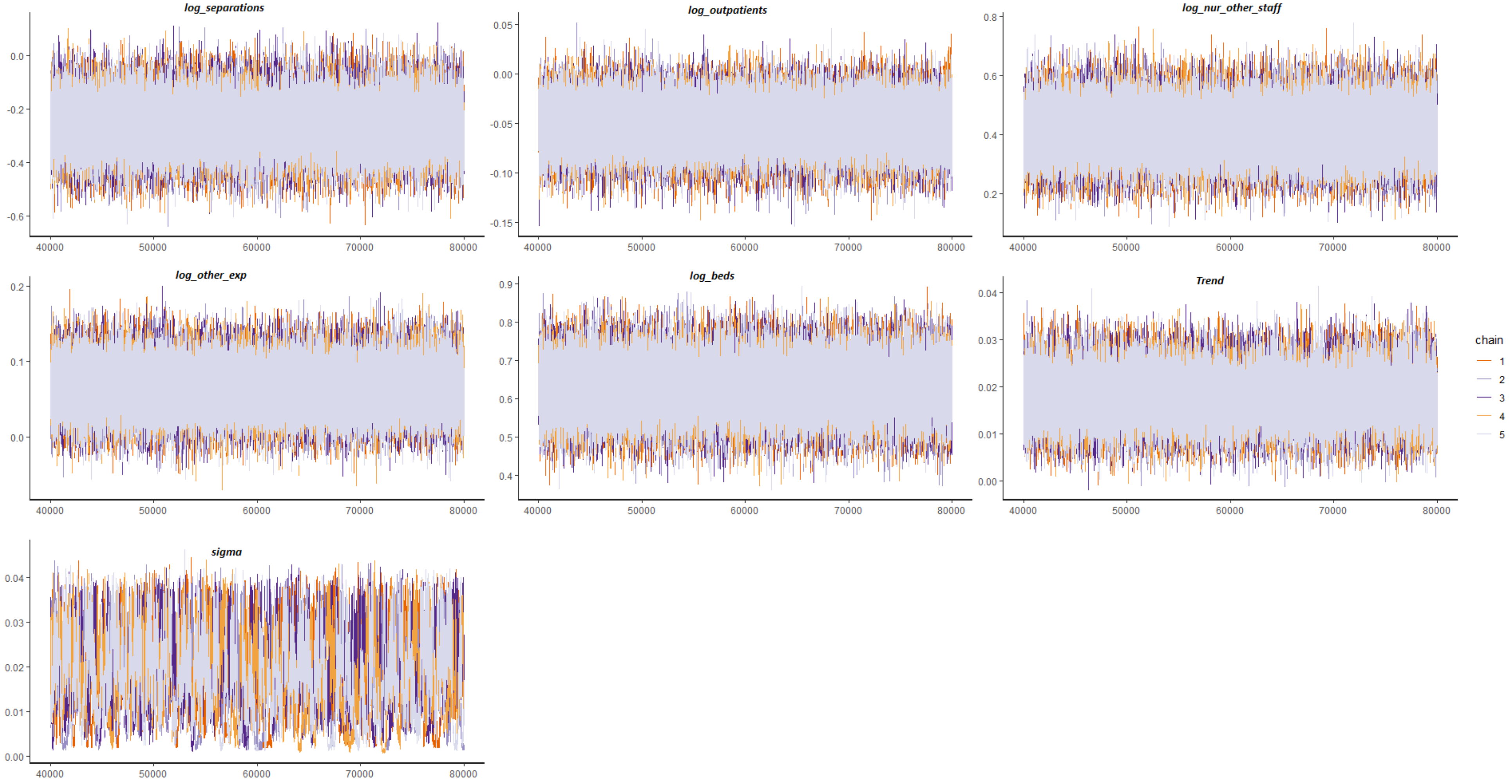
A1

Stan Code

```
data {
int<lower=0> N; // Number of observations
int<lower=0> J; // Number of groups
int<lower=0> P; // Number of independent variables
real Y[N]; // vector of dependent variables
matrix [N, P] X; // matrix of independent variables
int<lower=1, upper=J> state_id [N]; // vector specifying states and territories
}
parameters {
real<lower=0> sigma;
vector[P] beta;
real alpha;
vector[J] r; // vector of the variable that is a mixture of random effects and persistent technical inefficiencies
vector<lower=0, upper=1>[J] mix; // vector of mixing parameter
vector <lower = 0>[N]u; // vector of variable that captures transient technical inefficiencies
}
transformed parameters {
vector [N] yhat;
yhat =alpha + X*beta-u -r[state_id];
}
model {

// Priors
sigma~ gamma (1,1);
alpha ~ normal (0,5);
beta ~ normal (0,5);
u~ gamma (4,2);
mix ~ beta (4,3);

//Likelihoods
for (j in 1: J) {
  target += log_mix(mix[j],
                 normal_lpdf(r[j] | 0,5),
                 gamma_lpdf(r[j] |4,2)); // likelihood that specifies the mixture components
}
target += normal_lpdf (Y| yhat, sigma); // likelihood that specifies the input distance function
}

generated quantities {
vector[J] PE; // vector that collects persistent technical efficiency
vector[N] y_predict; // vector of data that could be used for posterior predictive checks
vector[N] log_lik; // vector of log-likelihood values
vector[N] TE; //vector that collects transient technical efficiency
for (j in 1: J)
PE[j] = exp (-((1-mix[j]) *r[j]));
for (i in 1: N) {
y_predict[i] = normal_rng (alpha + X[i]*beta-u[i]-r[state_id[i]], sigma);
log_lik[i] = normal_lpdf(Y[i] |alpha + X[i]*beta-u[i] -r[state_id[i]], sigma);
TE[i] = exp(-u[i]);
}
}
```

A2

Trace plot of first-order parameters and standard deviation



A3

Posterior Predictive Plot