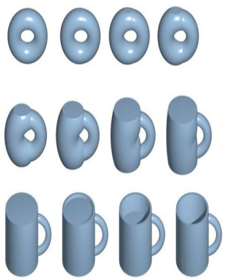
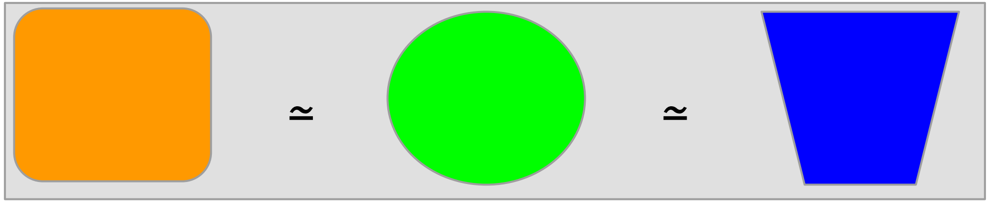
Introduction to Topology

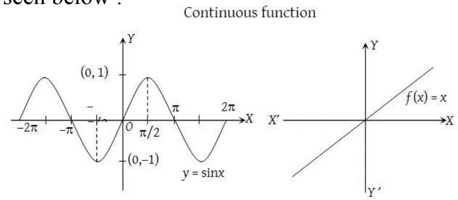


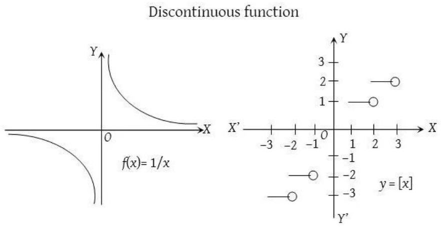
Topology comes from the Greek words τόπος, 'place, location', and λόγος, 'study', which derives to the study of the most basic properties of space. It is a branch of pure mathematics that defines objects by their characteristics that are unaffected by “continuous deformation” - an example of a continuous deformation is the image to the left; from a doughnut to a coffee mug. Topology begins as a highly visual and relatively intuitive area of mathematics, however it becomes quite rigorous, formal and challenging as the content becomes more general.The abstraction of known information differentiating objects from each other is what topology is built upon as it is seen as a core subject in modern abstract mathematics. Due to this fact, this topic is very broad, so I will mention only a few concepts fundamental in topology, such as continuous functions, fixed-points and topological invariants. One simple step of understanding topology is to know that although the 3 figures below are all geometrically unique, to a topologist these objects are actually all indistinguishable from one another.



Continuous Functions

Being continuous, mathematically means having no abrupt changes in values for any given function. Example functions that are continuous and discontinuous can be seen below :





A more intuitive understanding for continuous functions can come from the notions of Fixed points:

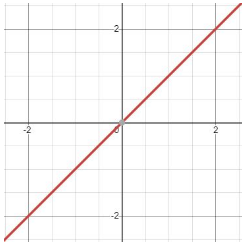
Fixed Points

A fixed point of a function is a point that “goes nowhere” during a transformation, formally stated as:

$f(x) = x.$

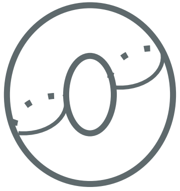
All this function is, is just the line:

$x = y.$ Shown below to the left.



So, for a function to be continuous, the curve must always cross this line where $f(x) = x$.

Topological Invariants



$V - E + F = \chi$:
$4 - 8 + 4 = 0$

$V - E + F = \chi$:
$2 - 4 + 4 = 2$

An example of a topological invariant is something called the Euler Characteristic, χ, which for objects that can continuously deform into one another, χ will have similar values. So we can understand that the two objects above, a torus and a sphere are not topologically equivalent.

References

Wikipedia contributors. (2020, November 5). *Topology*. Wikipedia. https://en.wikipedia.org/wiki/Topology#:%7E:text=In%20mathematics%2C%20topology%20(from%20the,but%20not%20tearing%20or%20gluing.