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Factor-Analysis-Based Directional Distance Function: The case of New Zealand hospitals

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Abstract

This paper develops a new factor-analysis-based (FAB) approach for choosing the optimal direction in a directional distance function (DDF) analysis. It has the combined merits of factor analysis and slacks-based measure (SBM) and incorporates the relative ease with which various input-output could be adjusted. This development relieves the dependency of price information that is normally unavailable in the provision of public goods. This new FAB-DDF model has been applied on a dataset containing all public hospitals in New Zealand (NZ) observed during 2011-2017. The empirical results indicate that the average reduction across different labor is in the range of 3-10 percent, and the corresponding figure for capital input is 25.7 percent. The case-adjusted inpatient-discharge and price-adjusted outpatient-visit are used as measures of desirable output, the average efficiencies are 92.7 percent and 99 percent respectively. Hospital readmission within 28 days of discharge is used as a measure for undesirable output, and the average efficiency score is 90 percent. These evidence support the suspicion that perverse incentives might exist under the National Health Targets abolished in 2018, which was a set of six indicators used in the last decade to evaluate the performance of local District Health Boards.

Keywords: factor-analysis-based measure; directional distance function; NZ hospital efficiency; hospital readmission

JEL classification: C61; D24; I11; I18

1. Introduction

As the outcome of the integration between distance function and gauge function, the directional distance function (DDF) analysis has gathered considerable momentum in production economics with empirical applications in a wide range of areas such as labor, health, energy and transport etc. Built upon the exploration work established by many studies (Shephard, 1970; Shephard and Färe, 1974; McFadden, 1978; Färe, 1988; and Luenberger, 1992, 1994, 1995), the first DDF was proposed by Chambers *et al.* (1996), and the analysis was further developed in Chung *et al.* (1997) and Chambers *et al.* (1998). Discussions about the theoretical properties of a DDF are available in these works as well as the modelling of bad (undesirable) outputs. However, little advancement has been made toward the choice of directions. A large number of applications arbitrarily set a fixed direction, which is either exclusively towards the reduction of bad output(s), or allowing for radical contraction in inputs at the same time. When it has been demonstrated that efficiency estimates may not be robust to the choice of directions in such analysis (Vardanyan and Noh, 2006; Agee *et al.* 2012; Pang and Deng, 2014), the issue of how to select the optimal direction emerged. For example, Pang and Deng (2014) find that the average efficiency of Chinese service sectors is above that of the industrial sectors if only reductions in sulfur dioxide and carbon dioxide emissions were considered. The conclusion would be reversed when labor, capital and energy inputs are allowed to be simultaneously contracted.

Alternative directions were suggested previously (Färe *et al.* 2005; Färe *et al.* 2006; Kumar, 2006; Lee *et al.* 2002 etc.), but none of these studies treats direction as an endogenous choice variable until recently (Färe *et al.* 2013; Zofio *et al.* 2013; and Atkinson and Tsionas, 2016). The idea is to find the optimal direction towards profit maximization through the construction of a profit function. The optimal choice in the direction of *marginal* profit maximization is further suggested by Lee (2014) and

Deng (2016).[1] Nonetheless, applications of the above techniques require data on prices. Given the difficulty of obtaining accurate information on price, especially the price of undesirable output, these developments are mainly theoretical with limited empirical attention. In addition, another constantly neglected aspect in performance studies is the ease at which inputs and outputs can be adjusted. To illustrate, for those inputs (or bad outputs) which can be easily reduced, the redundancy of these variables can happen immediately and therefore pose a lesser concern; on the contrary, for those that are more onerous to adjust, once committed, efficiency can only be improved in the long run and should be given more consideration in designing policies initiatives encouraging more efficient utilization of public resources. The new factor-analysis-based DDF model proposed in this study addresses this issue and the application does not require data on prices.

We first demonstrate that the selection of an optimal direction vector in a DDF analysis is equivalent to the choice of an optimal weight vector in a slacks-based measure (SBM) of efficiency. Next, we prove that the optimal weight vector in an SBM can be found using a factor-analysis-based approach.[2] Although both parametric method (Färe *et al.* 2005, 2006, 2012; Chambers *et al.* 2013; Feng and Serletis, 2014; Atkinson and Tsionas, 2016; Badau *et al.* 2016) and nonparametric method (Chung *et al.* 1997; Boyd *et al.* 2002; Zofio *et al.* 2013; Lee, 2014; Pang *et al.* 2015; Deng, 2016) are available to construct a DDF, this study focuses solely on the nonparametric technique.

In terms of empirical application, a newly available multifaceted administrative dataset is employed. This dataset contains all public hospitals managed by the 20 local District Health Boards (DHBs) in NZ during the period of 2011-2017.[3] The majority

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[1] Maximizing the *marginal* profit is considered to be a more practical approach compared to the conventional profit maximization because it involves a step-by-step improvement and ''wait-and-see'' decision process.

[2] For the principle of factor analysis method, refer to Boivin and Ng (2006), Foerster *et al.* (2011) and Johnson and Wichern (2013).

[3] Initially, there were 21 DHBs established in 2000, two of them were merged in 2010. The analysis in this study is built upon the stabilized post-merge period from 2011 to 2017. Profiles of the 20 DHBs are presented in ***Appendix 1***. They vary considerably in size, with Waitemata being the largest DHB serving over half a million

of health care systems worldwide face challenges imposed by tight public budgets, an aging population, and more chronic diseases. In 2017, total health expenditures in NZ amounts to $24.5 billion and 9.2% of GDP (OECD Health Statistics). Debates about system inefficiency resulted in a series of major structural changes since the 1990s (Ashton, 2005, 2009; Cumming *et al.* 2014 and Mays *et al.* 2013). In spite of this, sound performance measures are yet to be established for the health sector.

Healthcare services in NZ is mainly funded through tax. Public hospitals provide most of the secondary and tertiary healthcare services such as surgery, specialist treatments and emergency services. General practitioners, practice nurses, pharmacists and other health professionals working within a Primary Health Organization (PHO) are contracted by the government to provide primary healthcare services. District Health Boards (DHBs) were the local authorities responsible for providing health services to their geographically defined communities. DHBs own public hospitals as their provider arms and are funded by the Ministry of Health (MOH) through a population-based funding formula (PBFF).[4] Performance of the DHBs were monitored through quarterly assessment of the six indicators specified by the National Health Targets.[5] These targets are primarily partial output measures and there is no control for input usage. Many dimensions of healthcare services, such as acute hospital admissions and non-Emergency Department outpatient-visit, are completely unaccounted for. The degree to which the targets could create perverse incentives by diverting resources away from unmeasured services to measured ones were unknown. In other words, there were risks that the National Health Targets were achieved at the expenses of lowering overall productivity and efficiency. Many concern that the hospitals might, for instance, discharge acute patients sooner in order to accommodate more elective surgeries, leading to undesirable outcomes. This study employed the number of

population and West Coast being the smallest DHB with a population just over 30,000.

4 The PBFF allocates resources between DHBs based on a core model which assesses the relative healthcare needs of the local populations via historical average expenditure for different demographic groups. The PBFF does incorporate adjusters to account for factors such as populations with low access to healthcare services, rural areas and overseas visitors and refugees.

5 The National Health Targets are presented in ***Appendix 2***, they were first introduced in 2008 aiming to improve the performance of the health sector.

readmissions within 28 days of discharge as a measure of undesirable (bad) output.

The rest of the paper proceeds as follows. The next section discusses the dual correspondence between the directional distance function (DDF) and slacks-based measure (SBM) of efficiency. Section 3 develops the new FAB-DDF model and provides a step-by-step guide for application. The next section evaluates the efficiency of NZ public hospitals using this new approach and compares the results with those obtained under conventional DDFs and SBM. Section 5 concludes.

2. Dualities

2.1. The directional distance functions

Let $\boldsymbol{x} = (x_1, \cdots, x_N) \in \mathfrak{R}_+^N$ denotes the vector for inputs, $\boldsymbol{y} = (y_1, \cdots, y_M) \in \mathfrak{R}_+^M$ denotes a vector of desirable (good) outputs and $\boldsymbol{b} = (b_1, \cdots, b_B) \in \mathfrak{R}_+^B$ denotes a vector of undesirable (bad) outputs. The production possibility set which represents the technology describing the transformation of inputs into outputs is given by:

$$\mathbb{T}^{\mathbf{DDF}} = \{(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b})\text{: such that } \boldsymbol{x} \text{ can produce } (\boldsymbol{y}, \boldsymbol{b})\}. \tag{2.1}$$

One can refer to Chambers *et al.* (1996), Chung *et al.* (1997), and Färe *et al.* (2006) for the standard assumptions made on the technology.[6] A DDF is an alternative way to represent the technology from a computational viewpoint and can be described as:

$$\vec{D}(\boldsymbol{z}; \boldsymbol{g}) = \max\{\beta: (\boldsymbol{z} + \beta\boldsymbol{g})' = (\boldsymbol{x} + \beta\boldsymbol{g}_{\boldsymbol{x}}, \boldsymbol{y} + \beta\boldsymbol{g}_{\boldsymbol{y}}, \boldsymbol{b} + \beta\boldsymbol{g}_{\boldsymbol{b}}) \in \mathbb{T}^{\mathbf{DDF}}\}. \tag{2.2}$$

Here, $\boldsymbol{z} = (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b})'$ is the collective vector in $\mathfrak{R}_+^N \times \mathfrak{R}_+^M \times \mathfrak{R}_+^B$ containing all observed inputs and outputs. $\boldsymbol{g} = (\boldsymbol{g}_{\boldsymbol{x}}, \boldsymbol{g}_{\boldsymbol{y}}, \boldsymbol{g}_{\boldsymbol{b}})'$ is the vector of "directions" in which the observed inputs and outputs could be scaled. It is natural to specify that $\boldsymbol{g}_{\boldsymbol{x}} < \mathbf{0}$, $\boldsymbol{g}_{\boldsymbol{y}} > \mathbf{0}$ and $\boldsymbol{g}_{\boldsymbol{b}} < \mathbf{0}$. Thus, this function seeks the simultaneous maximum

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[6] Standard assumptions include: it is a convex, closed set; doing nothing is feasible; there is no free lunch; inputs and good outputs are freely disposable; good and bad outputs are null-joint and weakly disposable etc.

proportional reduction in $(\boldsymbol{x}, \boldsymbol{b})$ and expansion in $\boldsymbol{y}$. The distance is conventionally measured in a preassigned fixed direction to the boundary of $\mathbb{T}^{\mathbf{DDF}}$, for example, arbitrarily setting $\boldsymbol{g} = (-\boldsymbol{x}, \boldsymbol{y}, -\boldsymbol{b})'$. Common properties of the DDF include:

P1: Translation: $\vec{D}(\boldsymbol{z} + \tau\boldsymbol{g}; \boldsymbol{g}) = \vec{D}(\boldsymbol{z}; \boldsymbol{g}) - \tau$.
P2: Homogeneity of degree minus one in $\boldsymbol{g}$: $\vec{D}(\boldsymbol{z}; \tau\boldsymbol{g}) = \tau^{-1}\vec{D}(\boldsymbol{z}; \boldsymbol{g})$.
P3: Input monotonicity: $\vec{D}(\breve{\boldsymbol{x}}, \boldsymbol{y}, \boldsymbol{b}; \boldsymbol{g}) \geq \vec{D}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}; \boldsymbol{g})$ for $\breve{\boldsymbol{x}} \geq \boldsymbol{x}$.
P4: Good output monotonicity: $\vec{D}(\boldsymbol{x}, \breve{\boldsymbol{y}}, \boldsymbol{b}; \boldsymbol{g}) \leq \vec{D}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}; \boldsymbol{g})$ for $\breve{\boldsymbol{y}} \geq \boldsymbol{y}$.
P5: Bad output monotonicity: $\vec{D}(\boldsymbol{x}, \boldsymbol{y}, \breve{\boldsymbol{b}}; \boldsymbol{g}) \geq \vec{D}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}; \boldsymbol{g})$ for $\breve{\boldsymbol{b}} \geq \boldsymbol{b}$.
P6: Concavity: $\vec{D}(\boldsymbol{z}; \boldsymbol{g})$ is concave in $\boldsymbol{z} + \beta\boldsymbol{g} \in \mathbb{T}^{\mathbf{DDF}}$.
P7: Non-negativity: $\vec{D}(\boldsymbol{z}; \boldsymbol{g}) \geq 0$ if and only if $\boldsymbol{z} + \beta\boldsymbol{g} \in \mathbb{T}^{\mathbf{DDF}}$.

Property P1 states that if inputs and bad outputs are contracted by $\tau(|\boldsymbol{g}_x|, |\boldsymbol{g}_b|)$[7] and good outputs are expanded by $\tau\boldsymbol{g}_y$, then the value of the resulting distance function will be more efficient by the amount τ. Property P2 is the analog of P1, stating that changing the unit of direction vector does not change the relative size of inefficiency. Property P3 is a monotonicity property corresponding to strong disposability of inputs. It states that if a firm produces the same amount of desirable and undesirable outputs, but with more inputs, inefficiency will not *decrease*. Similarly, property P4 states that if desirable output increase, holding inputs and undesirable outputs constant, inefficiency does not *increase*. Property P5 is monotonicity with respect to bad output, increase in bad output will lead to non-decreasing inefficiency, holding inputs and desirable outputs constant. Property P6 is proven in Luenberger (1992) and is equivalent to assume the production possibility set $\mathbb{T}^{\mathbf{DDF}}$ is convex. Property P7 specifies that inefficiency is non-negative, zero inefficiency exists if and only if the observation is operating on the boundary of $\mathbb{T}^{\mathbf{DDF}}$, i.e. 100% efficient.

2.2. The slacks-based measure of efficiency

The production possibility set in a SBM of efficiency can be defined in a similar way as the following:

[7] Here, $|\cdot|$ is to obtain the absolute values for all elements of a vector.

$$\mathbb{T}^{\mathbf{SBM}} = \{(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b})\text{: such that } \boldsymbol{x} \text{ can produce } (\boldsymbol{y}, \boldsymbol{b})\}. \quad (2.3)$$

One can refer to Tone (2001) and Fukuyama and Weber (2009) for the standard assumptions made on the technology. A slacks-based measure of efficiency can be considered as the weighted sum of input excesses, bad output excesses and good output shortfalls, the corresponding description is specified as:

$$\overrightarrow{SBM}(\boldsymbol{z}; \boldsymbol{v}) = \max\{\boldsymbol{vs}: (\boldsymbol{z} + \boldsymbol{s})' = (\boldsymbol{x} + \boldsymbol{s_x}, \boldsymbol{y} + \boldsymbol{s_y}, \boldsymbol{b} + \boldsymbol{s_b}) \in \mathbb{T}^{\mathbf{SBM}}\}. \quad (2.4)$$

Here, $\boldsymbol{v} = (\boldsymbol{v_x}, \boldsymbol{v_y}, \boldsymbol{v_b}) \in \Re_+^N \times \Re_+^M \times \Re_+^B$ is the exogenous weights assigned to various dimensional slacks which are collectively represented by the vector $\boldsymbol{s} = (\boldsymbol{s_x}, \boldsymbol{s_y}, \boldsymbol{s_b})'$. Similar to analysis in DDF, it is common to consider that $\boldsymbol{v_x} < \mathbf{0}$, $\boldsymbol{v_y} > \mathbf{0}$, and $\boldsymbol{v_b} < 0$. Properties of the SBM of efficiency include:

D1: Translation: $\overrightarrow{SBM}(\boldsymbol{z} + \tau\boldsymbol{v}; \boldsymbol{v}) = \overrightarrow{SBM}(\boldsymbol{z}; \boldsymbol{v}) - \tau(\|\boldsymbol{v}\|_2)^2$.
D2: Homogeneity of degree one in $\boldsymbol{v}$: $\overrightarrow{SBM}(\boldsymbol{z}; \tau\boldsymbol{v}) = \tau\overrightarrow{SBM}(\boldsymbol{z}; \boldsymbol{v})$.
D3: Input monotonicity: $\overrightarrow{SBM}(\breve{\boldsymbol{x}}, \boldsymbol{y}, \boldsymbol{b}; \boldsymbol{v}) \geq \overrightarrow{SBM}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}; \boldsymbol{v})$ for $\breve{\boldsymbol{x}} \geq \boldsymbol{x}$.
D4: Good output monotonicity: $\overrightarrow{SBM}(\boldsymbol{x}, \breve{\boldsymbol{y}}, \boldsymbol{b}; \boldsymbol{v}) \leq \overrightarrow{SBM}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}; \boldsymbol{v})$ for $\breve{\boldsymbol{y}} \geq \boldsymbol{y}$.
D5: Bad output monotonicity: $\overrightarrow{SBM}(\boldsymbol{x}, \boldsymbol{y}, \breve{\boldsymbol{b}}; \boldsymbol{v}) \geq \overrightarrow{SBM}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}; \boldsymbol{v})$ for $\breve{\boldsymbol{b}} \geq \boldsymbol{b}$.
D6: Concavity: $\overrightarrow{SBM}(\boldsymbol{z}; \boldsymbol{v})$ is concave in $\boldsymbol{z} + \boldsymbol{s} \in \mathbb{T}^{\mathbf{SBM}}$.
D7: Non-negativity: $\overrightarrow{SBM}(\boldsymbol{z}; \boldsymbol{v}) \geq 0$ if and only if $\boldsymbol{z} + \boldsymbol{s} \in \mathbb{T}^{\mathbf{SBM}}$.

Property D1 states that if inputs and bad outputs are contracted by $\tau(|\boldsymbol{v_x}|, |\boldsymbol{v_b}|)$ and good outputs are expanded by $\tau\boldsymbol{v_y}$, then the value of the resulting slacks-based measure will be more efficient by the amount $\tau(\|\boldsymbol{v}\|_2)^2$. Property D2 implies that a change in the unit of the weight vector does not affect the relative size of inefficiency. Property D3 is a monotonicity property corresponding to strong disposability of inputs, inefficiency will not *decrease* for a firm produces the same amount of outputs with more inputs. Likewise, D4 and D5 are monotonicity properties with respect to desirable and undesirable output(s). Property D6 is equivalent to say that $\mathbb{T}^{\mathbf{SBM}}$ is convex and D7 indicates inefficiency cannot be negative.

2.3. The dual relationship between DDF and SBM

It is obvious that DDF and SBM share intuitively the same properties, the issue of choosing a particular direction towards the technological frontier is also one and the same (Chung *et al.* 1997; Chambers *et al.* 1998; Tone, 2001; Färe and Grosskopf, 2000; Färe *et al.* 2005; Fukuyama and Weber, 2009; and Hudgins and Primont, 2007). To illustrate the idea in a two-dimensional space, one could consider a simplified scenario with only one good output (y) and one bad output (b) for a *fixed* input vector $\boldsymbol{x}$, as presented in ***Figure 1***. The collective vector $\boldsymbol{g} = (\mathbf{0}, g_y, g_b)'$ represents the direction from an observed location A to a frontier location B; whereas in a slacks-based measure, the same direction from A to B would be represented by the weight vector $\boldsymbol{v} = (\mathbf{0}, v_y, v_b)$. Given an observation operating at point A, different directions will lead to different projection points onto the frontier, i.e. different Bs, therefore the distance from A to B will vary depends on the particular directional path being taken. The results from an efficiency benchmarking exercise won't be robust and comparable (Vardanyan and Noh, 2006; Agee *et al.* 2012; Pang and Deng, 2014; Atkinson and Tsionas, 2016) without an agreed direction.

[Insert ***Figure 1*** approximately here]

As illustrated by ***Figure 1***, $\tan\theta = g_y/|g_b|$ can be used to locate the projection direction in a DDF analysis. The larger g_y is relative to $|g_b|$, the closer the direction is to $\overrightarrow{\mathrm{AC}}$; conversely, the smaller g_y is relative to $|g_b|$, the closer the direction is to $\overrightarrow{\mathrm{AD}}$. In a SBM of efficiency, v_y and $|v_b|$ indicate the preference degree of objective function to reductions in good-output and bad-output slacks. One can view the weight vector $\boldsymbol{v}$ as same as the direction vector $\boldsymbol{g}$ (Fukuyama and Weber, 2009). The larger v_y is relative to $|v_b|$, the closer the direction is to $\overrightarrow{\mathrm{AC}}$ in a slacks-based measure; the smaller v_y is relative to $|v_b|$, the closer the direction would be to $\overrightarrow{\mathrm{AD}}$.

Provided that the choice of a direction vector $\boldsymbol{g}$ in a DDF analysis is equivalent to the choice of the weight vector $\boldsymbol{v}$ in a SBM of efficiency, let's consider a DDF specified as below:

$$\vec{D}(\boldsymbol{z};\boldsymbol{g}) = \max\{\beta: (\boldsymbol{z}+\beta\boldsymbol{g})' = (\boldsymbol{x}+\beta\boldsymbol{g_x}, \boldsymbol{y}+\beta\boldsymbol{g_y}, \boldsymbol{b}+\beta\boldsymbol{g_b}) \in \mathbb{T}^{\mathbf{DDF}}\} \tag{2.5}$$

$$\mathbb{T}^{\mathbf{DDF}} = \{(\boldsymbol{x},\boldsymbol{y},\boldsymbol{b}): \textstyle\sum_{k=1}^{K}(\rho^k\boldsymbol{x}^k) \le \boldsymbol{x};\ \sum_{k=1}^{K}(\rho^k\boldsymbol{y}^k) \ge \boldsymbol{y};\ \sum_{k=1}^{K}(\rho^k\boldsymbol{b}^k) \le \boldsymbol{b};\ \boldsymbol{\rho} = (\rho^1,\cdots,\rho^K) \ge 0, \|\boldsymbol{\rho}\|_1 = 1\}.$$

One can simultaneously specify a SBM as:

$$\overrightarrow{SBM}(\boldsymbol{z};\boldsymbol{v}) = \max\{\boldsymbol{vs}: (\boldsymbol{z}+\boldsymbol{s})' = (\boldsymbol{x}+\boldsymbol{s_x}, \boldsymbol{y}+\boldsymbol{s_y}, \boldsymbol{b}+\boldsymbol{s_b}) \in \mathbb{T}^{\mathbf{SBM}}\} \tag{2.6}$$

$$\mathbb{T}^{\mathbf{SBM}} = \{(\boldsymbol{x},\boldsymbol{y},\boldsymbol{b}): \textstyle\sum_{k=1}^{K}(\rho^k\boldsymbol{x}^k) \le \boldsymbol{x};\ \sum_{k=1}^{K}(\rho^k\boldsymbol{y}^k) \ge \boldsymbol{y};\ \sum_{k=1}^{K}(\rho^k\boldsymbol{b}^k) \le \boldsymbol{b};\ \boldsymbol{\rho} = (\rho^1,\cdots,\rho^K) \ge 0, \|\boldsymbol{\rho}\|_1 = 1\},$$

where K is the number of observations or decision making units (DMUs), $\boldsymbol{z}^k = (\boldsymbol{x}^k,\boldsymbol{y}^k,\boldsymbol{b}^k)'$ is the collective input and output vector for observation $k = 1,2,\cdots,K$. The sign of inequality in $\sum_{k=1}^{K}(\rho^k\boldsymbol{b}^k) \le \boldsymbol{b}$ is often replaced by equality (Boyd *et al.* 2002; Pang and Deng, 2014), but this makes no difference to the results of the analysis.

For a given direction vector $\boldsymbol{g}$, the specifications provided by (2.5) and (2.6) would be equivalent if the following two supplementary conditions have been imposed on the weight vector $\boldsymbol{v}$:

$$\boldsymbol{s} = (\boldsymbol{s_x},\boldsymbol{s_y},\boldsymbol{s_b})' \parallel \boldsymbol{g} = (\boldsymbol{g_x},\boldsymbol{g_y},\boldsymbol{g_b})', \tag{2.7}$$

$$\boldsymbol{vg} = 1. \tag{2.8}$$

Equation (2.7) means the collective slacks vector $\boldsymbol{s}$ and the direction vector $\boldsymbol{g}$ are parallel to each other. Equation (2.8) implies that the inner product of vector $\boldsymbol{v}'$ and $\boldsymbol{g}$ equals to unity, so the weight vector and direction vector cannot be vertical.[8] For a DDF with a particular direction vector $\boldsymbol{g}$, one can always find an equivalent SBM by incorporating (2.7) and (2.8) into the constraints of specifying the $\mathbb{T}^{\mathbf{SBM}}$. Conversely,

[8] Proof of equation (2.8) is provided in ***Appendix 3***.

for a SBM with a specific weight vector $\boldsymbol{v}$, one can always find the corresponding equivalent DDF by including the same constraints in $\mathbb{T}^{\mathbf{DDF}}$. In other words, the problem of setting a direction vector $\boldsymbol{g}$ in DDF can always be converted to choosing a weight vector $\boldsymbol{v}$ in SBM.

Table 1 presents eight common directional settings, where $\boldsymbol{g} = (\mathbf{0}, \boldsymbol{y}, -\boldsymbol{b})'$ is the most frequently adopted choice in the literature; $\boldsymbol{g} = (-\boldsymbol{x}, \mathbf{0}, \mathbf{0})'$, $\boldsymbol{g} = (\mathbf{0}, \boldsymbol{y}, \mathbf{0})'$ or $\boldsymbol{g} = (\mathbf{0}, \mathbf{0}, -\boldsymbol{b})'$ are equivalent to the conventional DEA analysis with either inputs orientation, good outputs orientation, or bad outputs orientation; $\boldsymbol{g} = \left(-\boldsymbol{\psi}_{\boldsymbol{x}} \odot \boldsymbol{x}, \boldsymbol{\psi}_{\boldsymbol{y}} \odot \boldsymbol{y}, -\boldsymbol{\psi}_{\boldsymbol{b}} \odot \boldsymbol{b}\right)'$ where $\|\boldsymbol{\psi}_{\boldsymbol{x}}\|_1 + \left\|\boldsymbol{\psi}_{\boldsymbol{y}}\right\|_1 + \|\boldsymbol{\psi}_{\boldsymbol{b}}\|_1 = 1$ is equivalent to the conventional SBM model (Fukuyama and Weber, 2009; Krüger, 2017), therefore it does not deal with the issue of finding the optimal weight vector in a SBM either. Moreover, the first three directional choices are generally considered to be exogenous (Chambers *et al.* 1996; Charnes *et al.* 1978; Färe *et al.* 2005, 2006; Feng and Serletis, 2014), which lack appropriate theoretical justification but the programs are relatively easy to solve. The rest considers the choice of direction as endogenous, and it is determined through common behavioral assumption such as (marginal) profit maximization. The resulting mathematical program is however more complicated to solve. This study attempts to find an optimal solution for endogenous directional choice without relying on price information but utilizes the potential correlation between observations.

[Insert ***Table 1*** approximately here]

3. The Factor-Analysis-Based Directional Distance Function (FAB-DDF)

3.1. The factor-analysis-based distance measure

Let us denote the distance from an observed location A to its projection onto the frontier location B as $\|B - A\| = \|\boldsymbol{s}\|$, where $\boldsymbol{s} = (\boldsymbol{s}_{\boldsymbol{x}}, \boldsymbol{s}_{\boldsymbol{y}}, \boldsymbol{s}_{\boldsymbol{b}})' \in \Re_+^N \times \Re_+^M \times \Re_+^B$ is the collective vector of slacks associated with all inputs and outputs, and $\boldsymbol{s_y} \geq \boldsymbol{0}$, $\boldsymbol{s_x} \leq \boldsymbol{0}$, and $\boldsymbol{s_b} \leq \boldsymbol{0}$. In the slacks-based measure specified by equation (2.6), this distance is provided by $\|\boldsymbol{s}\|_C = \boldsymbol{v} \cdot \boldsymbol{s}$ with an exogenously preassigned weight vector $\boldsymbol{v}$. For example, the slacks-based measure in Fukuyama and Weber (2009) sets the following:

$$\boldsymbol{v} = \left(\frac{-1}{2N}\frac{1}{x_1}, \cdots, \frac{-1}{2N}\frac{1}{x_N}, \frac{1}{2(M+B)}\frac{1}{y_1}, \cdots, \frac{1}{2(M+B)}\frac{1}{y_M}, \frac{-1}{2(M+B)}\frac{1}{b_1}, \cdots, \frac{-1}{2(M+B)}\frac{1}{b_B}\right). \tag{3.1}$$

Next let us consider two alternative measures of distance:

The Euclidean distance $\|\boldsymbol{s}\|_E = (\boldsymbol{s}'\boldsymbol{s})^{1/2}$, (3.2)

The Mahalanobis distance $\|\boldsymbol{s}\|_M = (\boldsymbol{s}'\boldsymbol{\Sigma}^{-1}\boldsymbol{s})^{1/2}$, (3.3)

with $\boldsymbol{\Sigma}$ represents the input-output covariance matrix.

The Mahalanobis distance is often preferred because it is unitless and scale-invariant through the incorporation of the correlation structure in the data set. However, it is a nonlinear combination of the distances (slacks) in various dimensions, the task of solving the corresponding nonlinear programming problem is a nontrivial one. Nonetheless, as a weighted distance it makes the slacks from various dimensions comparable. In other words, it reduces the multi-dimensional input-output vector $\boldsymbol{z} = (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b})' \in \Re_+^N \times \Re_+^M \times \Re_+^B$ into a one dimensional scalar measure.

The core idea behind the factor analysis is similar, which is the approximation of the covariance matrix $\boldsymbol{\Sigma}$ (Johnson and Wichern, 2013). A typical linear factor analysis equation can be specified as:

$$\boldsymbol{z}_{(N+M+B)\times 1} = \boldsymbol{\mu}_{(N+M+B)\times 1} + \boldsymbol{L}_{(N+M+B)\times Q}\boldsymbol{F}_{Q\times 1} + \boldsymbol{\varepsilon}_{(N+M+B)\times 1}, \tag{3.4}$$

where μ_i is the mean of variable i across all observations; $\boldsymbol{L}$ is the loading matrix; F_i is the ith common factor; $Q \leq N + M + B$ is the number of common factors; ε_i is the ith specific factor; $\boldsymbol{F}$ and $\boldsymbol{\varepsilon}$ are independent; $\mathbb{E}(\boldsymbol{F}) = \boldsymbol{0}$, $\mathrm{Cov}(\boldsymbol{F}) = \boldsymbol{I}$;

$\mathbb{E}(\boldsymbol{\varepsilon}) = \mathbf{0}$, $\text{Cov}(\boldsymbol{\varepsilon}) = \boldsymbol{\Psi}$, where $\boldsymbol{\Psi}$ is a diagonal matrix; the relationships of $\boldsymbol{\Sigma} = \text{Cov}(\boldsymbol{z}) = \boldsymbol{L}\boldsymbol{L}' + \boldsymbol{\Psi}$ and $\text{Cov}(\boldsymbol{z}, \boldsymbol{F}) = \boldsymbol{L}$ can be obtained easily, where the covariance matrix $\boldsymbol{\Sigma}$ is a nonsingular matrix.

There is information loss if $Q < N + M + B$ because the original data is of dimensions $N + M + B$, and the information loss can be measured by the specific factors. In order to figure out the weights for comparing distances in various dimensions, we set $Q = N + M + B$ instead of simply reducing the dimensions. Since $\boldsymbol{\Sigma}$ is nonsingular, one can always find $N + M + B$ common factors. Given $Q = N + M + B$, consider $\boldsymbol{\varepsilon} = \mathbf{0}$ and $\boldsymbol{\Psi} = \mathbf{0}$ for now, equation (3.4) can be simplified into the following:

$$\boldsymbol{z}_{(N+M+B)\times 1} = \boldsymbol{\mu}_{(N+M+B)\times 1} + \boldsymbol{L}_{(N+M+B)\times(N+M+B)}\boldsymbol{F}_{(N+M+B)\times 1}, \tag{3.5}$$

where $\boldsymbol{\Sigma} = \boldsymbol{L}\boldsymbol{L}'$. There are $N + M + B$ eigenvalues, referred to as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{N+M+B} \geq 0$, and their corresponding normalized eigenvectors are $\boldsymbol{e}_i$ $(i = 1, \cdots, N + M + B)$. The following can be obtained by factoring the covariance matrix:

$$\boldsymbol{\Sigma} = \lambda_1 \boldsymbol{e}_1 \boldsymbol{e}_1' + \cdots + \lambda_{N+M+B} \boldsymbol{e}_{N+M+B} \boldsymbol{e}_{N+M+B}'$$

$$= \left[\sqrt{\lambda_1}\boldsymbol{e}_1, \cdots, \sqrt{\lambda_{N+M+B}}\boldsymbol{e}_{N+M+B}\right] \begin{bmatrix} \sqrt{\lambda_1}\boldsymbol{e}_1' \\ \vdots \\ \sqrt{\lambda_{N+M+B}}\boldsymbol{e}_{N+M+B}' \end{bmatrix}. \tag{3.6}$$

From (3.6) and $\boldsymbol{\Sigma} = \boldsymbol{L}\boldsymbol{L}'$, we have:

$$\boldsymbol{L} = \left[\sqrt{\lambda_1}\boldsymbol{e}_1, \cdots, \sqrt{\lambda_{N+M+B}}\boldsymbol{e}_{N+M+B}\right]. \tag{3.7}$$

In summary, the sample means computed from the observed data $\boldsymbol{z}$ can be used to estimate $\boldsymbol{\mu}$; and the eigenvalues and eigenvectors of sample covariance matrix can be used to estimate $\boldsymbol{L}$; so the common factors $\boldsymbol{F} = (F_1, \cdots, F_{N+M+B})$ can be obtained by equation (3.5), that is, $\boldsymbol{F} = \boldsymbol{L}^{-1}(\boldsymbol{z} - \boldsymbol{\mu})$. Because $\boldsymbol{L}$ is composed by mutually perpendicular eigenvectors, its inverse matrix always exists.

When the units of the variables are not comparable, researchers usually standardize with $\tilde{\boldsymbol{z}} = (\frac{z_1-\bar{z}_1}{\sqrt{\sigma_{11}}}, \cdots, \frac{z_{N+M+B}-\bar{z}_{N+M+B}}{\sqrt{\sigma_{N+M+B,N+M+B}}})'$, where $\bar{z}_i$ and $\sqrt{\sigma_{ii}}$ are the sample mean and standard deviation of variable i. After standardization, $\tilde{\boldsymbol{\mu}} = \boldsymbol{0}$ and $\tilde{\boldsymbol{F}} = \tilde{\boldsymbol{L}}^{-1}\tilde{\boldsymbol{z}}$, where $\tilde{\boldsymbol{L}} = \left[\sqrt{\tilde{\lambda}_1}\tilde{\boldsymbol{e}}_1, \cdots, \sqrt{\tilde{\lambda}_{N+M+B}}\tilde{\boldsymbol{e}}_{N+M+B}\right]$ and $\tilde{\boldsymbol{\Sigma}} = \tilde{\boldsymbol{L}}\tilde{\boldsymbol{L}}'$ is the sample correlation matrix. Let's denote the element of matrix $\tilde{\boldsymbol{L}}^{-1}$ as α_{ij}. According to $\tilde{\boldsymbol{F}} = \tilde{\boldsymbol{L}}^{-1}\tilde{\boldsymbol{z}}$,

$$\begin{aligned}
\tilde{F}_1 &= \alpha_{11}\tilde{z}_1 + \alpha_{12}\tilde{z}_2 + \cdots + \alpha_{1,N+M+B}\tilde{z}_{N+M+B} \\
\tilde{F}_2 &= \alpha_{21}\tilde{z}_1 + \alpha_{22}\tilde{z}_2 + \cdots + \alpha_{2,N+M+B}\tilde{z}_{N+M+B} \\
&\vdots \\
\tilde{F}_{N+M+B} &= \alpha_{N+M+B,1}\tilde{z}_1 + \alpha_{N+M+B,2}\tilde{z}_2 + \cdots + \alpha_{N+M+B,N+M+B}\tilde{z}_{N+M+B}
\end{aligned} \tag{3.8}$$

As demonstrated in ***Appendix 4***, the variance contribution rate of factor $\tilde{F}_i$ is $\frac{\tilde{\lambda}_i}{N+M+B}$. And $\tilde{F}_T = \sum_{i=1}^{N+M+B} \frac{\tilde{\lambda}_i \tilde{F}_i}{N+M+B}$ represents a comprehensive score, which is a common dimensionality reduction strategy in the field of factor analysis. Considering the variance decomposition of factor $\tilde{F}_i$, we can obtain:

$$\mathrm{Var}(\tilde{F}_i) = \sum_{j=1}^{N+M+B} \alpha_{ij}\mathrm{Cov}(\tilde{z}_j, \tilde{F}_i) = 1.^{9} \tag{3.9}$$

Therefore, $\alpha_{ij}\mathrm{Cov}(\tilde{z}_j, \tilde{F}_i)$ indicates the importance of variable $\tilde{z}_j$ to factor $\tilde{F}_i$. Similarly, one can obtain $\mathrm{Var}(\tilde{F}_T) = \sum_{i=1}^{N+M+B} \frac{\tilde{\lambda}_i}{N+M+B}\mathrm{Cov}(\tilde{F}_i, \tilde{F}_T)$ considering the variance decomposition of $\tilde{F}_T$, i.e. $\sum_{i=1}^{N+M+B} \frac{\tilde{\lambda}_i}{N+M+B} \frac{\mathrm{Cov}(\tilde{F}_i,\tilde{F}_T)}{\mathrm{Var}(\tilde{F}_T)} = 1$. Therefore, $\frac{\tilde{\lambda}_i}{N+M+B} \frac{\mathrm{Cov}(\tilde{F}_i,\tilde{F}_T)}{\mathrm{Var}(\tilde{F}_T)}$ indicates the importance of factor $\tilde{F}_i$ to comprehensive score $\tilde{F}_T$.

Taking into account the importance of $\tilde{z}_j$ to factor $\tilde{F}_i$ and the importance of $\tilde{F}_i$ to comprehensive score $\tilde{F}_T$, $\sum_{i=1}^{N+M+B} \frac{\tilde{\lambda}_i \alpha_{ij}}{N+M+B} \frac{\mathrm{Cov}(\tilde{F}_i,\tilde{F}_T)\mathrm{Cov}(\tilde{z}_j,\tilde{F}_i)}{\mathrm{Var}(\tilde{F}_T)}$ indicates the importance

[9] View $\tilde{F}_i$ as an investment portfolio of $\tilde{z}_j$, $j = 1, \cdots, N + M + B$, then according to the capital asset pricing model $\mathrm{Cov}(\tilde{z}_j, \tilde{F}_i)$ is the Beta coefficient of $\tilde{z}_j$ that measures the systematic risk of $\tilde{z}_j$, and $\sum_{j=1}^{N+M+B} \alpha_{ij}\mathrm{Cov}(\tilde{z}_j, \tilde{F}_i)$ is the Beta coefficient of the investment portfolio.

of $\tilde{z}_j$ to $\tilde{F}_T$. Thus the weights $\sum_{i=1}^{N+M+B} \frac{\tilde{\lambda}_i \alpha_{ij}}{N+M+B} \frac{\mathrm{Cov}(\tilde{F}_i, \tilde{F}_T)\mathrm{Cov}(\tilde{z}_j, \tilde{F}_i)}{\mathrm{Var}(\tilde{F}_T)}$ $(j = 1, \cdots, N + M + B)$ can be used to weigh the distances ($|s_j|$) in various dimensions to obtain an average distance, that is:

$$\|\boldsymbol{s}\|_C = \boldsymbol{v} \cdot \boldsymbol{s} = \left(\cdots \sum_{i=1}^{N+M+B} \frac{\tilde{\lambda}_i \alpha_{ij}}{N+M+B} \frac{\mathrm{Cov}(\tilde{F}_i, \tilde{F}_T)\mathrm{Cov}(\tilde{z}_j, \tilde{F}_i)\mathrm{Sig}(s_j)}{\mathrm{Var}(\tilde{F}_T)} \cdots \right)_{(N+M+B)\times 1} \cdot \boldsymbol{s} \qquad (3.10)$$

where $\mathrm{Sig}(s_j) = -1$ if $s_j < 0$ and $\mathrm{Sig}(s_i) = +1$ if $s_j \geq 0$. The sum of the weights equals to 1, i.e. $\sum_{j=1}^{N+M+B} \sum_{i=1}^{N+M+B} \frac{\tilde{\lambda}_i \alpha_{ij}}{N+M+B} \frac{\mathrm{Cov}(\tilde{F}_i, \tilde{F}_T)\mathrm{Cov}(\tilde{z}_j, \tilde{F}_i)}{\mathrm{Var}(\tilde{F}_T)} = 1$.[10]

This weighting method, as specified by equation (3.10), has several advantages: (1) same as the Mahalanobis distance, it is unitless, scale-invariant, and accounts for the correlation structure of the data set; but (2) unlike the Mahalanobis distance and Euclidean distance, it is linear. The corresponding slacks-based measure is therefore easy to solve; (3) it uses common factors that are mutually perpendicular to explain the relative importance of variables (i.e. inputs and outputs). Due to the desirable statistical properties processed by the common factors, distances in different dimensions are comparable after adjusting the variables with the common factors; and (4) compared to conventional factor-based analysis, we set the number of common factors equal to the rank of the covariance matrix to prevent information loss, i.e. $\boldsymbol{\Sigma} = \boldsymbol{L}\boldsymbol{L}'$ (or $\tilde{\boldsymbol{\Sigma}} = \tilde{\boldsymbol{L}}\tilde{\boldsymbol{L}}'$).

One particular caveat is that even though relative distance is commonly used, we are not able to define the weighted distance of slacks as the following:

$$\|\tilde{\boldsymbol{z}}^{\boldsymbol{r}} \odot \boldsymbol{s}\|_C = \boldsymbol{v}(\tilde{\boldsymbol{z}}^{\boldsymbol{r}} \odot \boldsymbol{s}) = \sum_{j=1}^{N+M+B} \sum_{i=1}^{N+M+B} \frac{\tilde{\lambda}_i \alpha_{ij}}{N+M+B} \frac{\mathrm{Cov}(\tilde{F}_i, \tilde{F}_T)\mathrm{Cov}(\tilde{z}_j, \tilde{F}_i)\mathrm{Sig}(s_j)}{\mathrm{Var}(\tilde{F}_T)} \frac{s_j}{\tilde{z}_j}$$

---

[10] $\sum_{j=1}^{N+M+B} \sum_{i=1}^{N+M+B} \frac{\tilde{\lambda}_i}{N+M+B} \frac{\mathrm{Cov}(\tilde{F}_i, \tilde{F}_T)\alpha_{ij}\mathrm{Cov}(\tilde{z}_j, \tilde{F}_i)}{\mathrm{Var}(\tilde{F}_T)} = \sum_{i=1}^{N+M+B} \frac{\tilde{\lambda}_i}{N+M+B} \frac{\mathrm{Cov}(\tilde{F}_i, \tilde{F}_T)}{\mathrm{Var}(\tilde{F}_T)} \sum_{j=1}^{N+M+B} \alpha_{ij} \mathrm{Cov}(\tilde{z}_j, \tilde{F}_i) = 1.$

where $\tilde{\boldsymbol{z}}^{r} = \left(\frac{\mathbf{1}}{\tilde{z}_1}, \cdots, \frac{\mathbf{1}}{\tilde{z}_{N+M+B}}\right)'$ and $\odot$ denotes the Hadamard product of two vectors. This is because the standardized $\tilde{z}_j$ could be negative. The following can be specified instead:

$$\tilde{\boldsymbol{z}}^{rr} = \left(\frac{\sqrt{\sigma_{11}}}{z_1}, \cdots, \frac{\sqrt{\sigma_{N+M+B,N+M+B}}}{z_{N+M+B}}\right)', \tag{3.10}$$

and we can calculate:

$$\begin{aligned} \|\tilde{\boldsymbol{z}}^{rr} \odot \boldsymbol{s}\|_C &= \boldsymbol{v}(\tilde{\boldsymbol{z}}^{rr} \odot \boldsymbol{s}) \\ &= \sum_{j=1}^{N+M+B} \sum_{i=1}^{N+M+B} \frac{\tilde{\lambda}_i \alpha_{ij}}{N+M+B} \frac{\mathrm{Cov}(\tilde{F}_i,\tilde{F}_T)\mathrm{Cov}(\tilde{z}_j,\tilde{F}_i)\sqrt{\sigma_{jj}}\mathrm{Sig}(s_j)}{\mathrm{Var}(\tilde{F}_T)} \frac{s_j}{z_j}. \end{aligned} \tag{3.11}$$

$\|\tilde{\boldsymbol{z}}^{rr} \odot \boldsymbol{s}\|_C$ can be interpreted as using $\sum_{i=1}^{N+M+B} \frac{\tilde{\lambda}_i \alpha_{ij}}{N+M+B} \frac{\mathrm{Cov}(\tilde{F}_i,\tilde{F}_T)\mathrm{Cov}(\tilde{z}_j,\tilde{F}_i)\sqrt{\sigma_{jj}}\mathrm{Sig}(s_j)}{\mathrm{Var}(\tilde{F}_T)} \frac{1}{z_j}$ to weigh the absolute distance s_j, or using $\sum_{i=1}^{N+M+B} \frac{\tilde{\lambda}_i \alpha_{ij}}{N+M+B} \frac{\mathrm{Cov}(\tilde{F}_i,\tilde{F}_T)\mathrm{Cov}(\tilde{z}_j,\tilde{F}_i)\mathrm{Sig}(s_j)}{\mathrm{Var}(\tilde{F}_T)}$ to weigh the relative distance $\frac{\sqrt{\sigma_{jj}}s_j}{z_j}$.

3.2. The factor-analysis-based DDF with an endogenous direction

To sum up, the development of this new approach is based primarily on the utilization of factor analysis in determining the optimal weight vector $\boldsymbol{v}$. The resulting weighted distance, i.e. $\boldsymbol{v} \cdot \boldsymbol{s}$, is then used as the objective function in a SBM of efficiency. Because of the dualities between DDF and SBM, one can obtain the direction vector $\boldsymbol{g}$ corresponding to the optimal weight vector $\boldsymbol{v}$. The resulting DDF analysis is therefore called the FAB-DDF in which not only the selection of direction is endogenous but also the reliance on any exogenous settings or price information is unnecessary. The specific steps involved in the application of this FAB-DDF can be summarized below:

Step 1. Standardize all the input-output variables $\boldsymbol{z}$ by $\tilde{\boldsymbol{z}} = \left(\frac{z_1 - \bar{z}_1}{\sqrt{\sigma_{11}}}, \cdots, \frac{z_{N+M+B} - \bar{z}_{N+M+B}}{\sqrt{\sigma_{N+M+B,N+M+B}}}\right)'$;

Step 2. Determine the optimal weight vector $\boldsymbol{v}$ in the SBM using factor analysis:

2.1 use the standardized sample data to obtain the sample correlation matrix $\tilde{\boldsymbol{\Sigma}}$;

2.2 calculate the eigenvalues and eigenvectors of $\tilde{\boldsymbol{\Sigma}}$, i.e. $(\tilde{\lambda}_i;\ \tilde{\boldsymbol{e}}_i)$ *where* $i = 1, \cdots, N + M + B$;

2.3 calculate the inverse matrix of $\tilde{\boldsymbol{L}} = \left[\sqrt{\tilde{\lambda}_1}\tilde{\boldsymbol{e}}_1, \cdots, \sqrt{\tilde{\lambda}_{N+M+B}}\tilde{\boldsymbol{e}}_{N+M+B}\right]$ *to obtain the elements of matrix* $\tilde{\boldsymbol{L}}^{-1}$, i.e. α_{ij};

2.4 calculate the common factors $\tilde{F}_i$ $(i = 1, \cdots, N + M + B)$ *and comprehensive factor* $\tilde{F}_T$ *using the factor analysis approach; and*

2.5 obtain the distance weights, $v_j = \sum_{i=1}^{N+M+B} \frac{\tilde{\lambda}_i \alpha_{ij}}{N+M+B} \frac{\mathrm{Cov}(\tilde{F}_i, \tilde{F}_T)\mathrm{Cov}(\tilde{z}_j, \tilde{F}_i)\sqrt{\sigma_{jj}}\mathrm{Sig}(s_j)}{\mathrm{Var}(\tilde{F}_T)} \frac{1}{z_j}$.

Step 3. Solve the SBM model (i.e. equation (2.6)) using the weight vector $\boldsymbol{v}$ obtained from the previous step.

Step 4. Based on the results obtained, further solve for the optimal directional vector $\boldsymbol{g}$ using equations (2.7) and (2.8). Substituting this direction vector $\boldsymbol{g}$ into the conventional DDF analysis (i.e. equation (2.5)) with the standardized input-output vector $\tilde{\boldsymbol{z}}$, identical efficiency scores as to the SBM model can be obtained.

If one only cares about the efficiency scores, there is no need to proceed with *Step 4*, the purpose of which is to solve the direction selection problem in case the DDF analysis has been chosen as the preferred approach.

4. Efficiency Analysis of New Zealand Hospitals

4.1. Data sources and description

The data used in this study are provided by the Ministry of Health (MOH), which contains input information for each local DHB in the form of monthly financial statements during the year 2011-2017. We constructed four inputs (the number of full time equivalent (FTE) medical doctors, nurses, other staff, and capital), two good outputs (case-weighted inpatient discharges and price-weighted outpatient visits), and one bad output (readmission within 28 days of discharge) to implement this new FAB-DDF model.

A multiple-step procedure is followed to derive measures that can more accurately reflect input volumes. In the first stage, we estimate *the price of medical service* by taking the ratio of payments made to *employed* medical staff to the total FTE doctors on the payroll. There is no equivalent FTE counts for doctors who are *outsourced* from private practices so the monetary expenses on outsourced medical is often used for this purpose. The problem is that the *monthly expenditures on outsourced inputs* often contain negative values as a result of balancing the accounts, any input volume measures derived from such financial accounts are unlikely to represent the actual usage, and there is no way to ascertain this deviation. One solution is to aggregate the *monthly expenditures on outsourced medical* over the whole financial year. The FTE counts for *outsourced* medical can now be estimated by taking the ratio of this aggregate expenditures and the *price* of medical service (estimated in the first stage for employed medical), assuming both hired medical and outsourced medical doctors receive similar remuneration. The final FTE counts are the sum of *employed* medical and estimated *outsourced* medical.

The total FTE counts for nurses and other staff are derived in the same way. Other staff is a weighted sum of allied professional staff, support staff and management staff. The weights used are the expenditure shares for each category.

Capital is often more challenge to measure due to the lack of data to separate the flow of capital services from capital stock. The number of installed beds is a common proxy variable for capital input (Aletras *et al.* 2007; Ancarani *et al.* 2009; Brown, 2003; Chang *et al.* 2004; Friesner *et al.* 2013; Herr, 2008; Herr *et al.* 2011; Worthington, 2004). Unfortunately, that information for NZ DHB was not consistently collected. Others resort to use measures like depreciation (Marcinko and Hetico, 2012; Zelman *et al.* 2009) and capital charges (Parkin and Hollingsworth, 1997). Depreciation intends to measure the reduction in the value of capital assets and is calculated using the straight-line method (i.e. assets depreciate by the same percentage each year) in NZ. Capital charges is considered to be the best proxy because it reflects the opportunity cost of capital employed in public health services (NZ Productivity Commission, 2017).

Output information is extracted from the National Minimum Hospital Datasets (NMDS) and National Non-Admitted Patient Collection (NNPAC) by the MOH. Two desirable output measures are used to reflect the full range of hospital services provided: case-weighted inpatient discharges and price-weighted outpatient visits. As mentioned previously, public hospitals in NZ are run and owned by DHBs to provide a variety of publicly funded health and disability services, they can be broadly categorized into inpatient admissions and outpatient visits. Although detailed case information is available for both categories (such as maternity, medical and surgical cases), the use of which comes at the cost of losing more degrees of freedom in such a small census dataset. Provided inpatient discharges have been adjusted using the case-mix methodology which accounts for the complexity of the diagnosis as well as the relative resources for treatment, the resulting output measures are reasonably comparable across different hospitals in different DHBs (Fraser and Nolan, 2017). Outpatient visits have been weighted with national prices (from the National Cost Collection and Pricing Programme) which are calculated for the purpose of inter-district flows. There are potentially two measures of undesirable outputs: adverse events[11] and readmissions. The former is identified and reported by each DHB on a voluntary basis. which implies it does not count as an objective measure comparable across observations. As a result, we considered hospital readmissions within 28 days of discharge as an indicator for undesirable output.

The final dataset is a balanced panel containing 20 observations (all DHBs) each year, for the year 2011-2017. Descriptive statistics of the variables for each DHB are presented in ***Table 2***. There are three DHBs in the city of Auckland serving over one third of the national population together, they are Counties Manukau DHB, Waitemata DHB, and Auckland DHB. Counties Manukau has the highest average number of

[11] Adverse events are cases that involve serious harm or death. They are collected by the Health Quality & Safety Commission and categorized into (1) harm from falls; (2) clinical management events such as delays in treatment, concerns about the accuracy of diagnosis, inadequate patient monitoring; and (3) medical overdose and surgical site infections.

outpatient-visit (29,923). Auckland generates the highest average number of inpatient-discharge (130,053), the associated undesirable outcomes are also the highest (77 self-reported adverse events and 14,786 readmissions).

[Insert ***Table 2*** approximately here]

4.2. Efficiency evaluation

Direction selection

During the first step of implementing this new FAB-DDF, the collective input-output vector $\boldsymbol{z}$ is standardized by $\tilde{\boldsymbol{z}} = \left(\frac{z_1-\bar{z}_1}{\sqrt{\sigma_{11}}}, \cdots, \frac{z_7-\bar{z}_7}{\sqrt{\sigma_{77}}}\right)'$ and the sample correlation matrix $\tilde{\boldsymbol{\Sigma}}$ is obtained. The Kaiser-Meyer-Olkin measure of sampling adequacy is 0.865, and the Bartlett's test of sphericity is significant at 0.1%. These indicate that the factor analysis based on this dataset is adequate. There are 7 eigenvalues of $\tilde{\boldsymbol{\Sigma}}$: $\tilde{\lambda}_1 = 6.5318$, $\tilde{\lambda}_2 = 0.2731$, $\tilde{\lambda}_3 = 0.0930$, $\tilde{\lambda}_4 = 0.0474$, $\tilde{\lambda}_5 = 0.0379$, $\tilde{\lambda}_6 = 0.0097$, and $\tilde{\lambda}_7 = 0.0072$. The corresponding eigenvectors being selected are:

$$(\tilde{\boldsymbol{e}}_1\ \tilde{\boldsymbol{e}}_2\ \tilde{\boldsymbol{e}}_3\ \tilde{\boldsymbol{e}}_4\ \tilde{\boldsymbol{e}}_5\ \tilde{\boldsymbol{e}}_6\ \tilde{\boldsymbol{e}}_7) = \begin{bmatrix} 0.3837 & 0.2017 & 0.2897 & 0.1670 & 0.6378 & 0.5156 & 0.1662 \\ 0.3828 & -0.2451 & 0.4593 & -0.1718 & -0.2097 & -0.3350 & 0.6298 \\ 0.3864 & -0.0501 & 0.1988 & -0.3992 & -0.4907 & 0.4892 & -0.4113 \\ 0.3570 & 0.7452 & -0.2857 & 0.2534 & -0.3604 & -0.0915 & 0.1819 \\ 0.3666 & -0.5786 & -0.3863 & 0.5791 & -0.1463 & 0.1556 & 0.0258 \\ 0.3880 & 0.0281 & 0.2767 & 0.1970 & 0.1985 & -0.5682 & -0.6091 \\ 0.3802 & -0.0765 & -0.5985 & -0.5872 & 0.3431 & -0.1646 & 0.0422 \end{bmatrix}.$$

The matrix $\tilde{\boldsymbol{L}} = \left[\sqrt{\tilde{\lambda}_1}\tilde{\boldsymbol{e}}_1, \cdots, \sqrt{\tilde{\lambda}_{N+M+B}}\tilde{\boldsymbol{e}}_{N+M+B}\right]$ can be obtained and its inverse matrix is:

$$\tilde{\boldsymbol{L}}^{-1} = \begin{bmatrix} 0.1501 & 0.1499 & 0.1512 & 0.1397 & 0.1434 & 0.1517 & 0.1488 \\ 0.3862 & -0.4688 & -0.0959 & 1.4260 & -1.1072 & 0.0533 & -0.1462 \\ 0.9492 & 1.5076 & 0.6511 & -0.9364 & -1.2663 & 0.9070 & -1.9630 \\ 0.7675 & -0.7889 & -1.8341 & 1.1638 & 2.6599 & 0.9046 & -2.6973 \\ 3.2760 & -1.0776 & -2.5189 & -1.8519 & -0.7517 & 1.0198 & 1.7620 \\ 5.2440 & -3.4082 & 4.9769 & -0.9315 & 1.5814 & -5.7765 & -1.6742 \\ 1.9578 & 7.4260 & -4.8518 & 2.1439 & 0.3028 & -7.1767 & 0.4992 \end{bmatrix}.$$

For the second step, the following weight vector in the SBM is obtained:

$$\boldsymbol{v} = \left(-\frac{0.1470\sqrt{\sigma_{x_1x_1}}}{x_1}, -\frac{0.1464\sqrt{\sigma_{x_2x_2}}}{x_2}, -\frac{0.1491\sqrt{\sigma_{x_3x_3}}}{x_3}, -\frac{0.1282\sqrt{\sigma_{x_4x_4}}}{x_4}, \frac{0.1347\sqrt{\sigma_{y_1y_1}}}{y_1}, \frac{0.1502\sqrt{\sigma_{y_2y_2}}}{y_2}, -\frac{0.1444\sqrt{\sigma_{b_1b_1}}}{b_1}\right)$$

where x_1, x_2, x_3, x_4, y_1, y_2 and b_1 correspond to the number of FTE medical staff, nurses, other staff, capital charges, outpatient visits, inpatient discharges and readmissions.

According to prior discussions, solving equations (2.7) and (2.8) will give us the direction vector $\boldsymbol{g}$ in a DDF analysis that corresponds to the weight vector $\boldsymbol{v}$ found in the SBM. For example, the weight vector for the largest DHB Waitemata, in year 2017, is

$$\boldsymbol{v} = (-0.0641,\ -0.0538,\ -0.0497,\ -0.0504, 0.0399, 0.0638,\ -0.0385),$$

and its equivalent direction vector is

$$\boldsymbol{g} = (-0.6403,\ -1.9533,\ -4.7692,\ -5.8545, 0.8679, 0.0000,\ -7.4479)'.$$

Because of limited space, we do not list the vectors of $\boldsymbol{v}$ and $\boldsymbol{g}$ associated with all observations, they are available upon request to the authors.

Discussion of Efficiency Estimates

Using the optimal weight vector $\boldsymbol{v}$ obtained above and solving the SBM programming problem (2.6), efficiency scores are computed based on this newly developed FAB-DDF model and the results are displayed in ***Table 3*** and ***Figure 2***.[12]

[Insert ***Table 3*** and ***Figure 2*** approximately here]

Wairarapa is the only DHB operating at full efficiency, while the other 19 DHBs have varying degrees of inefficiency. Over the entire sample period from 2011 to 2017, the average efficiency score is 91 percent. The rankings across different DHBs are quite robust in general when the results are compared with those obtained under conventional approaches, as displayed in ***Figure 2***. More specifically, the results under the FAB-DDF model closely resembles those from (i) the SBM model in which

[12] $efficiency = 1 - inefficiency = 1 - \overrightarrow{SBM}(\boldsymbol{z};\boldsymbol{v})$.

$\boldsymbol{v} = (\frac{-1}{8}\frac{1}{x_1}, \cdots, \frac{-1}{8}\frac{1}{x_4}, \frac{1}{6}\frac{1}{y_1}, \frac{1}{6}\frac{1}{y_2}, \frac{-1}{6}\frac{1}{b_1})$; (ii) the input orientated DEA with $\boldsymbol{g} = (-\boldsymbol{x}, \boldsymbol{0}, \boldsymbol{0})'$; (iii) the good-output orientated DEA with $\boldsymbol{g} = (\boldsymbol{0}, \boldsymbol{y}, \boldsymbol{0})'$; (iv) the bad-output orientated DEA with $\boldsymbol{g} = (\boldsymbol{0}, \boldsymbol{0}, -\boldsymbol{b})'$; and (v) multi-dimensional orientated DDFs with $\boldsymbol{g} = (\boldsymbol{0}, \boldsymbol{y}, -\boldsymbol{b})'$ and $\boldsymbol{g} = (-\boldsymbol{x}, \boldsymbol{y}, -\boldsymbol{b})'$.

To investigate the main sources of inefficiency, we further decompose the overall efficiency score obtained under the FAB-DDF model into various dimensional efficiency components and the results are presented in ***Table 4***.

[Insert ***Table 4*** approximately here]

The main sources of inefficiency comes from the utilization of capital and other staff, as well as the control of bad output (readmissions). The average efficiencies associated with capital input (eff_{cap}) and readmission reduction (eff_{readm}) are 74.3 and 89.9 percent, respectively, which are much lower than the efficiencies for providing good outputs. The average efficiency score for inpatient-discharge is 99.1 percent, with the majority have been operating close to full efficiency. These results are expected given the focus on the volume of elective discharges without other controls in the National Health Targets against which performance of NZ DHBs are monitored.

To achieve full efficiency, the amount of FTE medical staff, nurse, and all other staff can be decreased by 8 percent, 3 percent and 10 percent, respectively on average. The greatest source of input inefficiency is capital, as measured by the capital charges, an average of 25.7 percent downscale has been estimated. With respect to output(s), the largest inefficiency is an average of 10 percent reduction in readmissions.

For each DHB, ***Table 4*** provides an optimal direction to improve efficiency. For example, the Canterbury DHB should mostly focuse on capital utilization; the West Coast DHB could expand on the provision of good outputs (particularly inpatient-discharge) while keeping its capital charges in check, some might argue this is because of the small rural community West Coast DHB serves, i.e. caused by a shortage of demand instead of inefficient supply. The situation for the second smallest DHB (Tairawhiti) however is more complex, besides the extremely low capital efficiency score, it also needs to deal with excess in medical staff and inadequate provision of outpatient service.[13] Full efficiency is achieved for inpatient discharges so that Tairawhiti would be judged to have better performance under the National Health Targets although other dimensions, as well as the overall efficiency score, are much worse compared to the West Coast.

Shadow price analysis

As shown in ***Figure 2***, the conventional SBM of efficiency produces similar estimates as to the new FAB-DDF model. Meanwhile, the efficiency estimates obtained from conventional DDF analysis with the following direction vectors $\boldsymbol{g} = (-\boldsymbol{x}, \boldsymbol{0}, \boldsymbol{0})'$, $(\boldsymbol{0}, \boldsymbol{y}, \boldsymbol{0})'$, $(\boldsymbol{0}, \boldsymbol{y}, -\boldsymbol{b})'$, and $(-\boldsymbol{x}, \boldsymbol{y}, -\boldsymbol{b})'$ are higher, and the estimates from $\boldsymbol{g} = (\boldsymbol{0}, \boldsymbol{0}, -\boldsymbol{b})'$ are lower on average.

To illustrate the advantage of the new FAB-DDF approach, we compute the shadow price for medical staff. Under desirable theoretical properties, the salary paid to medical doctors should reflect its shadow price. Summary statistics of the average annual salary to medical staff are presented in ***Table 2***. For a given level of efficiency and using the implicit differentiation rule:

$$\frac{\partial z_i}{\partial z_j} = -\frac{\partial \overrightarrow{SBM}(\tilde{\mathbf{z}};\boldsymbol{v})/\partial \tilde{z}_j}{\partial \overrightarrow{SBM}(\tilde{\mathbf{z}};\boldsymbol{v})/\partial \tilde{z}_i} \sqrt{\sigma_{z_i z_i}} \Big/ \sqrt{\sigma_{z_j z_j}},$$

the shadow price of z_j with respect to z_i is derived. By solving the FAB-DDF

[13] Since capital charges is used as the proxy for the flow of capital services, it is possible that inefficiency in capital input might reflect the underlying fact that some public hospitals are being over charged. Capital charges is supposed to capture the opportunity cost of capital used in public services, i.e. what will it cost if the same capital is leased to the private sector? Nonetheless, it is highly questionable whether or not public use of capital is equivalent to private use given the obvious externalities associated with public health services. It has been reported that the NZ hospitals are experiencing problems in replacing and upgrading critical infrastructures.

model, one can obtain $\partial \overrightarrow{SBM}(\tilde{\mathbf{z}}; \boldsymbol{v})/\partial \tilde{z}_j$ and $\partial \overrightarrow{SBM}(\tilde{\mathbf{z}}; \boldsymbol{v})/\partial \tilde{z}_i$. A vast amount of literature treats good-output as z_i. Hence, the shadow price of medical staff can be measured by:

$\frac{\partial inpatient\_discharge}{\partial medical\_staff\_FTE}$ or $\frac{\partial outpatient\_visit}{\partial medical\_staff\_FTE}$.

Because the units of measurement for outpatient-visit and inpatient-discharge are not in dollar terms, the nominal shadow price can be approximated as:

$$costs\_per\_discharge \times \frac{\partial inpatient\_discharge}{\partial medical\_staff\_FTE},$$

where the costs per discharge is obtained from the total expenditures on all personnel, outsourced clinical services and clinical supplies, the summary statistics are provided in ***Table 2***. The correlation coefficients between the estimated shadow prices and the observed salary made to medical staff are displayed in ***Table 5***. The shadow prices obtained under the new FAB-DDF approach always correlate more closely and significantly with the observed salary payment than those from other approaches.

[Insert ***Table 5*** approximately here]

Furthermore, the efficiency decomposition displayed in ***Table 4*** indicates that the major source of output inefficiency is readmission, the control of which has been left out by the National Health Targets. The shadow values, as explained in Färe *et al.* (2006), and the total *shadow costs* associated with readmission are given by:

$$readmission \times costs\_per\_discharge \times \frac{\partial inpatient\_discharge}{\partial readmission}.$$

Since each DHB needs to reduce the number of readmission by $s_{b_1}\sqrt{\sigma_{b_1 b_1}}/b_1$ in order to achieve full efficiency in the bad-output dimension, the total costs association with *readmission reduction* can be calculated as:

$$s_{b_1}\sqrt{\sigma_{b_1 b_1}}/b_1 \times shadow\ costs\_readmission.$$

The variations in shadow costs and reduction costs are presented in ***Figure 3***, the top panel provides the average figure for each DHB during 2011-2017 and the bottom panel provides the national annual average. The largest DHB, Waitemata (WTM), has the highest readmission reduction costs (around $105 million) and the second highest shadow costs (around $291 million). Canterbury (CAN) has the highest shadow cost (around $330 million), but it is efficient in minimizing readmissions. West Coast DHB (WTC) is the third one down the list facing high shadow costs associated with readmission. On average, the reduction costs of readmission is $10 million over the last 7 years, implying substantial gain in health care efficiency from one of the missing component in the National Health Targets. The national average spikes in 2011 and 2016 (but falls in the years that follow) reassure this observation, which are the two years the issue of healthcare quality was brought up by the Heath Quality & Safety Commission through the release of public reports.

[Insert ***Figure 3*** approximately here]

5. Conclusions

As DDF analysis becomes increasingly popular in estimating production technologies involving multiple-output and multiple-input, the issue of justifying the choice of appropriate directions becomes crucial for robust efficiency estimates. This study proposes a new approach, namely, factor-analysis-based DDF (FAB-DFF), which provides a non-radial endogenous optimal direction for efficiency improvement. It is built upon the fundamental equivalence embedded within DDF and SBM.

Compared to conventional DDF and SBM analysis, this new FAB-DDF model has a number of merits. First of all, as a non-radical measure it can help eliminate any potential bias introduced by radial measures (Fukuyama and Weber, 2009). Second, the directional choice is endogenous rather than exogenous. Pre-fixed exogenous directions do not consider the relative importance of distances (slacks) in each dimension. However, the FAB-DDF takes the advantages of the Mahalanobis distance and factor analysis through incorporating the correlation structure of the input-output dataset, i.e. the relative ease with which various input-output can be adjusted. Finally, unlike the endogenous directions put forward by Zofio *et al.* (2013), Atkinson and Tsionas (2016), Lee (2014), and Deng (2016), this new FAB-DDF model does not rely on additional price information. A step-by-step guide for application of this FAB-DDF model has been provided and a dataset containing all the public hospitals in NZ observed during 2011-2017 is utilized.

The efficiency scores computed using the new approach for NZ hospitals are compared with those obtained under conventional DDF and SBM analysis (with preassigned directions). The SBM provides similar efficiency estimates but the shadow price analysis shows that the estimates from the new FAB-DDF approach are more closely correlated with the observed salary paid to medical doctors. To achieve full efficiency, on average, the number of FTE medical staff, nurse, other staff, capital charges, and readmission within 28 days of discharge need to be reduced by 8.3%, 3.1%, 10.1%, 25.7%, and 10.1%, respectively; and the number of price-weighted outpatient-visit and case-weighted inpatient-discharge can be increased by 7.3% and 0.9%. The main source of inefficiency comes from the utilization of capital input, followed by readmission reduction.[14] On average, the costs associated with reducing the number of readmission over the last 7 years is around $10 million, implying that substantial gain in healthcare efficiency is possible if the mission components in the National Health Targets could be built into future policy initiatives governing the evaluation of hospital performance.

[14] The final ranking of 20 District Health Boards in NZ is: Wairarapa (1st), Waikato (2nd), Bay of Plenty (3rd), Counties Manukau (4th), Auckland (5th), Lakes (6th), Canterbury (7th), Capital Coast (8th), Southern (9th), South Canterbury (10th), Nelson Marlborough (11th), Hawke's Bay (12th), West Coast (13th), Hutt Valley (14th), Taranaki (15th), MidCentral (16th), Northland (17th), Waitemata (18th), Whanganui (19th), Tairawhiti (20th) DHBs.

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Table 1: Common directions in a DDF analysis

Literature	Direction vector	Require Price information
Chambers *et al.* (1996), Kumar (2006)	$\boldsymbol{g} = (\boldsymbol{0}, \boldsymbol{y}, -\boldsymbol{b})'$	No
Charnes *et al.* (1978)	$\boldsymbol{g} = (-\boldsymbol{x}, \boldsymbol{0}, \boldsymbol{0})'$, $\boldsymbol{g} = (\boldsymbol{0}, \boldsymbol{y}, \boldsymbol{0})'$ or $\boldsymbol{g} = (\boldsymbol{0}, \boldsymbol{0}, -\boldsymbol{b})'$	No
Färe *et al.* (2005), Färe *et al.* (2006), Feng and Serletis (2014)	$\boldsymbol{g} = (\boldsymbol{0}, \boldsymbol{1}, -\boldsymbol{1})'$	No
Färe *et al.* (2013)	$\boldsymbol{g} = \left(\boldsymbol{0}, \frac{\boldsymbol{s_y}}{\Vert\boldsymbol{s_y}\Vert_1 + \Vert\boldsymbol{s_b}\Vert_1}, \frac{\boldsymbol{s_b}}{\Vert\boldsymbol{s_y}\Vert_1 + \Vert\boldsymbol{s_b}\Vert_1}\right)'$	No
Krüger (2017)	$\boldsymbol{g} = (-\boldsymbol{\psi_x} \odot \boldsymbol{x}, \boldsymbol{\psi_y} \odot \boldsymbol{y}, -\boldsymbol{\psi_b} \odot \boldsymbol{b})'$ where $\Vert\boldsymbol{\psi_x}\Vert_1 + \Vert\boldsymbol{\psi_y}\Vert_1 + \Vert\boldsymbol{\psi_b}\Vert_1 = 1$	No
Zofio *et al.* (2013), Atkinson and Tsionas (2016)	$(\boldsymbol{g_x}, \boldsymbol{g_y}, \boldsymbol{g_b})' = \arg \max_{\boldsymbol{g_x}, \boldsymbol{g_y}, \boldsymbol{g_b}} \{\pi\}$	Yes
Lee (2014)	$(\boldsymbol{g_y}, \boldsymbol{g_b})' = \arg \max_{\boldsymbol{g_y}, \boldsymbol{g_b}} \{\partial\pi/\partial\boldsymbol{x}\}$, $\boldsymbol{g_x} = \boldsymbol{0}$	Yes
Deng (2016)	$(\boldsymbol{g_y}, \boldsymbol{g_b})' = \arg \max_{\boldsymbol{g_y}, \boldsymbol{g_b}} \{\partial\pi/\partial c\}$, $\boldsymbol{g_x} = \boldsymbol{0}$	Yes

Source: authors' compliance.

$\odot$ denotes the Hadamard product of two vectors.

$\max_{\boldsymbol{g_x}, \boldsymbol{g_y}, \boldsymbol{g_b}}\{\pi\}$ means that enterprises choose optimal direction to maximize profit.

$\max_{\boldsymbol{g_y}, \boldsymbol{g_b}}\{\partial\pi/\partial\boldsymbol{x}\}$ means that enterprises choose optimal direction to maximize marginal profit of unit input.

$\max_{\boldsymbol{g_y}, \boldsymbol{g_b}}\{\partial\pi/\partial c\}$ means that enterprises choose optimal direction to maximize marginal profit of unit investment.

Table 2: Summary of input-output variables during 2011-2017

DHB	Outpatient Visits	Inpatient Discharges	Adverse Events	Readmissions	Medical staff (FTE)	Nurses (FTE)	Other staff (FTE)	Capital Charges (in $1000)	Salary to medical staff (in $1000)	Total adjusted costs per discharge (in $1000)
Auckland	**24,205**	**130,053**	**77**	**14,786**	**1,636**	**3,374**	**1,474**	**37,282**	**184.73**	**8.63**
	(1,764)	**(4,796)**	**(16)**	**(835)**	**(97)**	**(120)**	**(61)**	**(3,818)**	**(13.09)**	**(0.44)**
Bay of Plenty	14,294	38,537	12	5,301	323	1,118	430	6,326	208.36	7.54
	(920)	(2,723)	(2)	(461)	(22)	(42)	(26)	(1,174)	(6.94)	(0.17)
Canterbury	25,387	92,574	54	9,544	950	3,542	1,258	13,809	194.85	8.31
	(2,088)	(4,281)	(10)	(947)	(51)	(158)	(57)	(4,122)	(9.77)	(0.32)
Capital Coast	17,655	65,528	21	7,150	795	2,059	749	8,364	165.84	8.26
	(805)	(3,240)	(4)	(360)	(83)	(88)	(16)	(1,311)	(4.04)	(0.23)
Counties Manukau	**29,923**	**84,317**	**47**	**11,072**	**983**	**2,589**	**909**	**14,708**	**175.64**	**8.24**
	(1,265)	**(3,183)**	**(15)**	**(531)**	**(57)**	**(117)**	**(41)**	**(2,702)**	**(11.25)**	**(0.44)**
Hawke's Bay	10,458	27,152	12	3,789	312	858	376	4,193	165.40	8.44
	(656)	(762)	(4)	(324)	(22)	(40)	(10)	(1,557)	(9.38)	(0.55)
Hutt Valley	10,585	23,130	8	3,207	251	729	354	6,207	192.87	8.52
	(506)	(1,131)	(2)	(555)	(13)	(26)	(12)	(1,196)	(7.70)	(0.19)
Lakes	6,986	17,440	10	3,200	170	485	208	3,410	202.81	7.65
	(324)	(1,169)	(4)	(349)	(14)	(21)	(10)	(671)	(7.31)	(0.14)
MidCentral	11,606	28,133	19	3,845	308	960	422	8,529	196.40	9.02
	(564)	(791)	(2)	(205)	(20)	(29)	(8)	(1,776)	(5.21)	(0.51)
Nelson Marlborough	9,746	21,402	13	2,638	191	644	454	6,104	238.26	9.60
	(1,203)	(582)	(12)	(330)	(10)	(9)	(8)	(1,085)	(11.98)	(0.41)
Northland	10,243	27,483	13	4,277	272	973	421	7,926	210.38	9.23
	(974)	(1,326)	(6)	(136)	(30)	(45)	(14)	(1,788)	(5.80)	(0.56)

Table 2-continued: Summary of input-output variables during 2011-2017

DHB	Outpatient Visits	Inpatient Discharges	Adverse Events	Readmissions	Medical staff (FTE)	Nurses (FTE)	Other staff (FTE)	Capital Charges (in $1000)	Salary to medical staff (in $1000)	Total adjusted costs per discharge (in $1000)
South Canterbury	4,427	8,604	12	1,173	67	327	105	620	271.23	9.25
	(276)	(200)	(6)	(103)	(6)	(6)	(4)	(117)	(15.23)	(0.63)
Southern	19,508	52,084	41	6,151	519	1,593	632	8,607	222.01	8.37
	(2,397)	(1,761)	(12)	(658)	(23)	(51)	(9)	(1,679)	(8.24)	(0.39)
Tairawhiti	3,102	7,388	5	900	77	268	140	2,269	270.98	10.71
	(192)	(226)	(2)	(48)	(4)	(14)	(3)	(493)	(24.91)	(0.60)
Taranaki	7,859	17,588	9	2,881	154	571	245	5,826	206.92	9.04
	(419)	(762)	(6)	(411)	(5)	(22)	(8)	(733)	(16.93)	(0.30)
Waikato	23,412	82,131	41	10,845	725	2,384	986	15,568	209.94	8.17
	(1,455)	(4,089)	(9)	(1,448)	(52)	(124)	(39)	(1,800)	(6.74)	(0.31)
Wairarapa	3,559	6,174	5	890	49	212	87	487	234.02	8.56
	(425)	(214)	(3)	(142)	(4)	(16)	(6)	(167)	(15.25)	(0.45)
Waitemata	**25,550**	**72,621**	**43**	**14,509**	**877**	**2,579**	**1,150**	**17,122**	**181.55**	**9.31**
	(4,154)	**(6,568)**	**(10)**	**(1,885)**	**(74)**	**(194)**	**(60)**	**(4,546)**	**(7.87)**	**(0.30)**
West Coast	3,227	3,783	8	418	60	320	138	746	298.77	19.04
	(195)	(183)	(4)	(58)	(5)	(12)	(13)	(116)	(22.16)	(0.72)
Whanganui	4,924	11,157	9	1,972	114	391	155	1,801	218.65	8.73
	(364)	(287)	(4)	(202)	(4)	(10)	(4)	(372)	(5.47)	(0.52)

Table 3: Average efficiency estimates over 2011-2017

DHB	FAB-DDF	SBM	DDFs $(-\boldsymbol{x},\mathbf{0},\mathbf{0})'$	$(\mathbf{0},\boldsymbol{y},\mathbf{0})'$	$(\mathbf{0},\mathbf{0},-\boldsymbol{b})'$	$(\mathbf{0},\boldsymbol{y},-\boldsymbol{b})'$	$(-\boldsymbol{x},\boldsymbol{y},-\boldsymbol{b})'$
Auckland	0.980	0.979	1.000	1.000	0.985	1.000	1.000
	(0.027)	(0.027)	(0.000)	(0.000)	(0.021)	(0.000)	(0.000)
Bay of Plenty	0.996	0.996	0.999	0.998	0.986	1.000	1.000
	(0.007)	(0.007)	(0.003)	(0.003)	(0.034)	(0.001)	(0.000)
Canterbury	0.973	0.973	0.992	0.997	0.985	0.997	1.000
	(0.046)	(0.047)	(0.020)	(0.009)	(0.029)	(0.008)	(0.000)
Capital Coast	0.965	0.962	0.983	0.982	0.960	0.986	0.994
	(0.032)	(0.034)	(0.030)	(0.024)	(0.044)	(0.018)	(0.012)
Counties Manukau	0.984	0.985	1.000	0.993	0.972	0.997	1.000
	(0.020)	(0.020)	(0.000)	(0.009)	(0.041)	(0.006)	(0.000)
Hawke's Bay	0.894	0.896	0.955	0.927	0.768	0.938	0.985
	(0.054)	(0.053)	(0.025)	(0.053)	(0.117)	(0.045)	(0.015)
Hutt Valley	0.875	0.878	0.979	0.938	0.732	0.942	0.994
	(0.058)	(0.057)	(0.032)	(0.031)	(0.122)	(0.029)	(0.016)
Lakes	0.975	0.974	0.998	0.998	0.894	0.998	1.000
	(0.024)	(0.026)	(0.003)	(0.004)	(0.104)	(0.004)	(0.001)
MidCentral	0.831	0.832	0.922	0.833	0.714	0.870	0.969
	(0.027)	(0.028)	(0.082)	(0.030)	(0.026)	(0.019)	(0.040)
Nelson Marlborough	0.899	0.905	0.996	0.972	0.877	0.975	1.000
	(0.075)	(0.071)	(0.009)	(0.024)	(0.101)	(0.022)	(0.000)
Northland	0.810	0.806	0.893	0.814	0.636	0.858	0.965
	(0.026)	(0.029)	(0.076)	(0.061)	(0.018)	(0.055)	(0.034)
South Canterbury	0.958	0.959	0.990	0.988	0.939	0.989	0.998
	(0.053)	(0.052)	(0.018)	(0.021)	(0.085)	(0.020)	(0.005)
Southern	0.964	0.962	0.974	0.980	0.936	0.985	0.990
	(0.037)	(0.039)	(0.027)	(0.021)	(0.063)	(0.016)	(0.010)
Tairawhiti	0.744	0.747	0.964	0.962	0.804	0.982	0.990
	(0.018)	(0.017)	(0.054)	(0.066)	(0.025)	(0.031)	(0.018)
Taranaki	0.845	0.842	0.986	0.923	0.637	0.951	0.997
	(0.016)	(0.018)	(0.024)	(0.030)	(0.044)	(0.048)	(0.008)
Waikato	0.997	0.997	1.000	1.000	1.000	1.000	1.000
	(0.007)	(0.008)	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)
Wairarapa	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Waitemata	0.806	0.808	0.966	0.877	0.556	0.940	1.000
	(0.028)	(0.031)	(0.066)	(0.097)	(0.058)	(0.088)	(0.000)
West Coast	0.888	0.892	1.000	0.994	0.909	0.998	1.000
	(0.091)	(0.087)	(0.000)	(0.011)	(0.095)	(0.006)	(0.000)
Whanganui	0.805	0.806	0.875	0.851	0.581	0.882	0.946
	(0.015)	(0.016)	(0.025)	(0.030)	(0.055)	(0.057)	(0.029)
Total	0.909	0.910	0.974	0.951	0.844	0.964	0.991
	(0.087)	(0.086)	(0.049)	(0.069)	(0.160)	(0.055)	(0.020)

In the SBM model, weight vector is the one used by Fukuyama and Weber (2009).

Table 4: Decomposition of average efficiency during 2011-2017

DHB	eff	eff_{med}	eff_{nur}	eff_{oth}	eff_{cap}	eff_{outp}	eff_{inp}	eff_{readm}
Auckland	0.980	0.988	0.996	0.974	0.946	0.961	1.000	0.987
Bay of Plenty	0.996	1.000	0.998	0.997	0.989	0.996	1.000	0.993
Canterbury	0.973	0.989	0.990	0.987	0.872	0.958	1.000	1.000
Capital Coast	0.965	0.950	0.997	0.987	0.908	0.897	1.000	1.000
Counties Manukau	0.984	0.971	0.996	0.997	0.954	0.988	1.000	0.980
Hawke's Bay	0.894	0.844	0.990	0.880	0.657	0.952	1.000	0.907
Hutt Valley	0.875	0.842	1.000	0.841	0.606	0.992	0.998	0.820
Lakes	0.975	0.952	1.000	0.999	0.978	0.977	1.000	0.918
MidCentral	0.831	0.848	0.930	0.835	0.518	0.879	1.000	0.766
Nelson Marlborough	0.899	0.951	1.000	0.716	0.666	0.976	1.000	0.967
Northland	0.810	0.931	0.894	0.803	0.497	0.790	1.000	0.709
South Canterbury	0.958	0.916	0.960	0.983	0.939	0.956	1.000	0.952
Southern	0.964	0.996	1.000	0.993	0.806	0.932	1.000	0.996
Tairawhiti	0.744	0.660	0.945	0.721	0.252	0.623	1.000	0.933
Taranaki	0.845	0.955	0.992	0.867	0.386	0.946	1.000	0.713
Waikato	0.997	1.000	1.000	1.000	1.000	0.982	1.000	1.000
Wairarapa	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Waitemata	0.806	0.893	0.867	0.697	0.601	0.932	0.997	0.634
West Coast	0.888	0.936	0.859	0.869	0.777	0.949	0.819	1.000
Whanganui	0.805	0.720	0.975	0.832	0.514	0.849	1.000	0.707
Total	0.909	0.917	0.969	0.899	0.743	0.927	0.991	0.899

eff is the average efficiency for each DHB over 2011-2017, obtained under the FAB-DDF model.

$eff_{med} = \overline{\left(1 - s_{x_1}\sqrt{\sigma_{x_1x_1}}/x_1\right)}$ is the average efficiency for medical staff.

$eff_{nur} = \overline{\left(1 - s_{x_2}\sqrt{\sigma_{x_2x_2}}/x_2\right)}$ is the average efficiency for the nurse input.

$eff_{oth} = \overline{\left(1 - s_{x_3}\sqrt{\sigma_{x_3x_3}}/x_3\right)}$ is the average efficiency for all other staff.

$eff_{cap} = \overline{\left(1 - s_{x_4}\sqrt{\sigma_{x_4x_4}}/x_4\right)}$ is the average efficiency for capital input.

$eff_{outp} = \overline{\left(1 - s_{y_1}\sqrt{\sigma_{y_1y_1}}/y_1\right)}$ is the average efficiency for outpatient-visit.

$eff_{inp} = \overline{\left(1 - s_{y_2}\sqrt{\sigma_{y_2y_2}}/y_2\right)}$ is the average efficiency for inpatient-discharge.

$eff_{readm} = \overline{\left(1 - s_{b_1}\sqrt{\sigma_{b_1b_1}}/b_1\right)}$ is the average efficiency for readmissions.

$eff = 0.1470 \cdot eff_{med} + 0.1464 \cdot eff_{nur} + 0.1491 \cdot eff_{oth} + 0.1282 \cdot eff_{cap} + 0.1347 \cdot eff_{outp} + 0.1502 \cdot eff_{inp} + 0.1444 \cdot eff_{readm}.$

Table 5: Correlation coefficients between observed salary of medical staff and estimated shadow prices

Measures of Shadow Price for Medical Doctors	FAB-DDF	Conventional SBM	Conventional DDF with $\boldsymbol{g} = (\mathbf{0}, \boldsymbol{y}, \mathbf{0})'$
$\frac{\partial inpatient\_discharge}{\partial medical\_staff\_FTE}$	0.5279**	0.5127***	0.0247
	(0.0000)	(0.0000)	(0.7764)
$costs\_per\_discharge \times \frac{\partial inpatient\_discharge}{\partial medical\_staff\_FTE}$	0.5635***	0.5305***	0.0496
	(0.0000)	(0.0000)	(0.5681)

The values in parentheses are the significance of Pearson's correlation test.

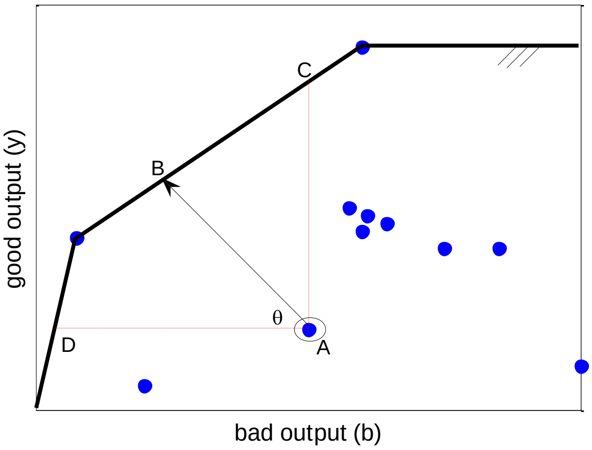

Figure 1 Direction selection in DDF and SBM

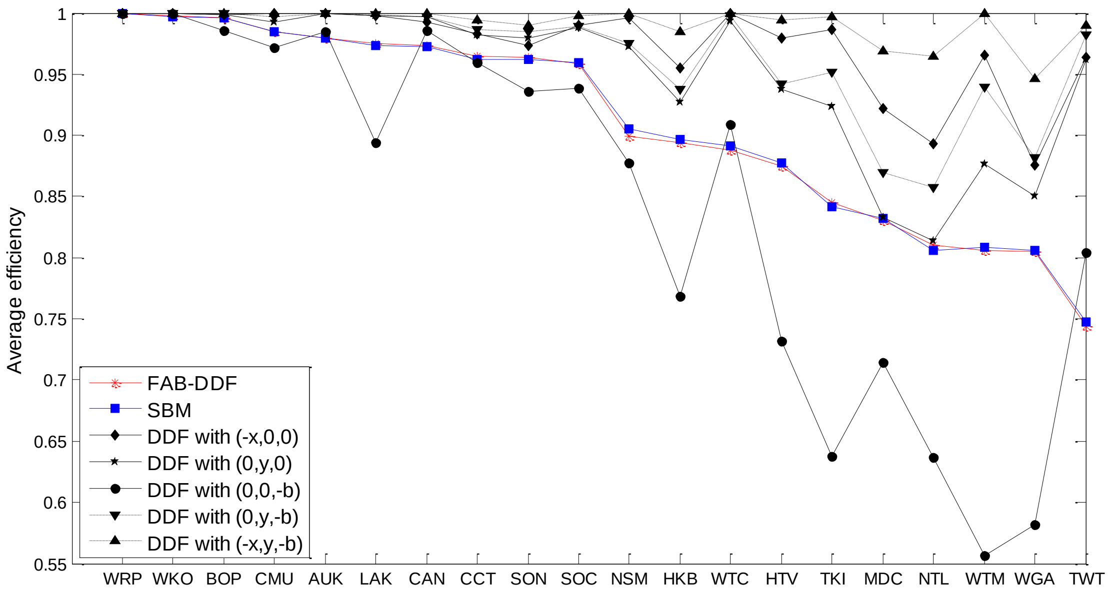

Figure 2 Average efficiency of NZ DHBs (2011-2017)

Wairarapa DHB: WRP; Waikato DHB: WKO; Bay of Plenty DHB: BOP; Counties Manukau DHB: CMU; Auckland DHB: AUK; Lakes DHB: LAK; Canterbury DHB: CAN; Capital Coast DHB: CCT; Southern DHB: SON; South Canterbury DHB: SOC; Nelson Marlborough DHB: NSM; Hawke's Bay DHB: HKB; West Coast DHB: WTC; Hutt Valley DHB: HTV; Taranaki DHB: TKI; MidCentral DHB: MDC; Northland DHB: NTL; Waitemata DHB: WTM; Whanganui DHB: WGA; Tairawhiti DHB: TWT.

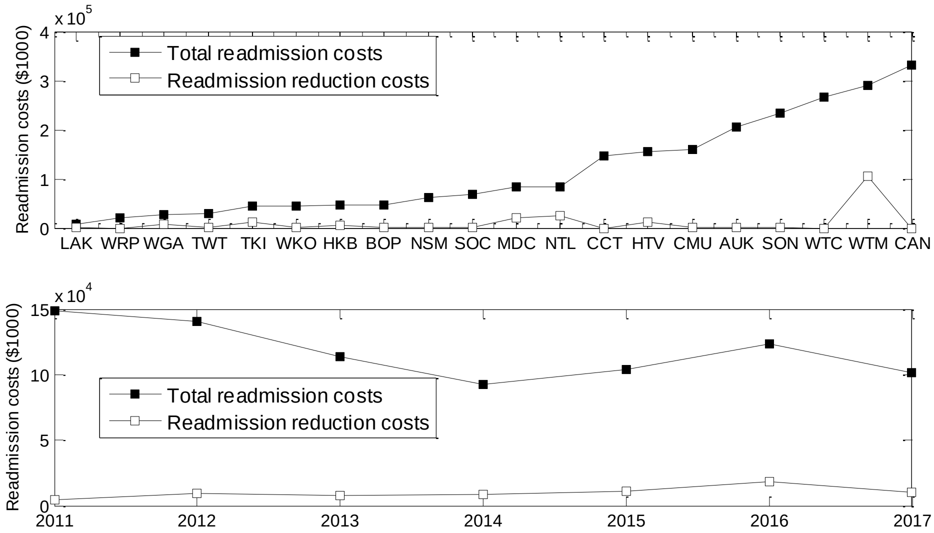

Figure 3 Readmission costs of NZ DHBs over 2011-2017

Calculated by the authors. Lakes DHB: LAK; Wairarapa DHB: WRP; Whanganui DHB: WGA; Tairawhiti DHB: TWT; Taranaki DHB: TKI; Waikato DHB: WKO; Hawke's Bay DHB: HKB; Bay of Plenty DHB: BOP; Nelson Marlborough DHB: NSM; South Canterbury DHB: SOC; MidCentral DHB: MDC; Northland DHB: NTL; Capital Coast DHB: CCT; Hutt Valley DHB: HTV; Counties Manukau DHB: CMU; Auckland DHB: AUK; Southern DHB: SON; West Coast DHB: WTC; Waitemata DHB: WTM;. Canterbury DHB: CAN.

Appendix 1: New Zealand DHBs

DHBs	Hospitals owned by each DHB	Population
Auckland	Auckland City Hospital; and Starship Children’s Hospital.	460,000
Bay of Plenty	Tauranga Hospital; and Whakatane Hospital.	220,000
Canterbury	Christchurch Hospital; Christchurch Women’s Hospital; Burwood Hospital; The Princess Margaret Hospital; Ashburton Hospital; and Hillmorton Hospital.	501,425
Capital Coast	Wellington Hospital; and Kenepuru Hospital.	300,000
Counties Manukau	Middlemore Hospital; Manukau Super Clinic and Surgery Centre.	512,130
Hawke's Bay	Hawke's Bay Hospital.	150,000
Hutt Valley	Hutt Hospital.	140,000
Lakes	Rotorua Hospitals; and Taupo Hospital.	108,000
MidCentral	Palmerston North Hospital; and Horowhenua Health Centre.	166,000
Nelson Marlborough	Nelson Hospital and Wairau Hospital.	134,500
Northland	Whangarei Hospital; Bay of Islands Hospital; Dargaville Hospital; and Kaitaia Hospital.	154,700
South Canterbury	Timaru Hospital.	55,626
Southern	Dunedin Hospital; Wakari Hospital; Lake district Hospital; and Southland Hospital.	315,000
Tairawhiti	Gisborne Hopsital.	46,000
Taranaki	Taranaki Base Hospital; and Hawera Hospital.	110,000
Waikato	Waikato Hospital.	360,000
Wairarapa	Wairarapa Hospital	40,000
Waitemata	North Shore Hospital; and Waitakere Hospital	560,000
West Coast	Grey base Hospital	31,000
Whanganui	Whanganui Hospital	60,120


Northland
Waitemata
Auckland
Counties Manukau
Bay of Plenty
Waikato
Tairawhiti
Lakes
Taranaki
Hawke's Bay
Whanganui
Capital and Coast
MidCentral
Hutt
Wairarapa
Nelson Marlborough
West Coast
Canterbury
(includes Chatham Island)
South Canterbury
Southern

Appendix 2: National Health Targets for 2017

Shorter Stays in Emergency Departments	95% of patients will be admitted, discharged, or transferred from an emergency department within six hours.
Improved Access to Elective Surgery	The volume of elective surgery will be increased by an average of 4000 discharges per year nationally. Each DHB is expected to meet the agreed number of elective surgeries annually.
Faster Cancer Treatment	85% of patients receive their first cancer treatment (or other management) within 62 days of being referred with a high suspicion of cancer and a need to be seen within 2 weeks.
Increased Immunisation	95% of 8-months-olds will have their primary course of immunisation (6 weeks, 3 months and 5 months immunisation events) on time.
Better Help for Smokers to Quit	90% of PHO enrolled patients who smoke have been offered help to quit smoking by a health care practitioner in the last 15 months.
Raising Healthy Kids	95% of obese children identified in the B4 School Check programme will be offered a referral to a health professional for clinical assessment and family-based nutrition, activity and lifestyle interventions by December 2017.

Appendix 3: proof of equation (2.8)

To make (2.5) equivalent to (2.6), the objective functions in (2.5) and (2.6) should be equal, therefore,

$$(\boldsymbol{vs})^2 = \boldsymbol{s}'\boldsymbol{s}(\boldsymbol{g}'\boldsymbol{g})^{-1}. \quad \text{(A1)}$$

By condition (2.7) in main text, $\boldsymbol{s} = \tau\boldsymbol{g}$, where τ is a constant, substituting this relationship into (A1), obtaining:

$$\tau^2(\boldsymbol{vg})^2 = \tau^2\boldsymbol{g}'\boldsymbol{g}(\boldsymbol{g}'\boldsymbol{g})^{-1}. \quad \text{(A2)}$$

Simplifying this equation, we obtain $\boldsymbol{vg} = 1$.

Appendix 4: derivation of the variance contribution rate of factors

The variance of variable z_i $(i = 1, \cdots, N + M + B)$ is $\mathrm{Var}(z_i)$, the total variance of all the variables is $\mathrm{tr}(\boldsymbol{\Sigma}) = \mathrm{Var}(z_1) + \mathrm{Var}(z_2) + \cdots + \mathrm{Var}(z_{N+M+B})$, where $\mathrm{Var}(z_i) = l_{i1}^2 + l_{i2}^2 + \cdots + l_{i,N+M+B}^2$ because $\boldsymbol{\Sigma} = \boldsymbol{LL}'$ and $L \equiv \left(l_{ij}\right)_{(N+M+B)\times(N+M+B)}$. Therefore, the contribution of the jth common factor F_j to $\mathrm{tr}(\boldsymbol{\Sigma})$ is $\sum_{i=1}^{N+M+B} l_{ij}^2$.

According to $\boldsymbol{L} = \left[\sqrt{\lambda_1}\boldsymbol{e}_1, \cdots, \sqrt{\lambda_{N+M+B}}\boldsymbol{e}_{N+M+B}\right]$ mentioned in the main text,

$$\sum_{i=1}^{N+M+B} l_{ij}^2 = \left(\sqrt{\lambda_j}\boldsymbol{e}_j\right)' \sqrt{\lambda_j}\boldsymbol{e}_j = \lambda_j \boldsymbol{e}_j' \boldsymbol{e}_j, \tag{A3}$$

where $\boldsymbol{e}_j'\boldsymbol{e}_j = 1$, thereby $\sum_{i=1}^{N+M+B} l_{ij}^2 = \lambda_j$.

Therefore, the variance contribution rate of factor F_j to $\mathrm{tr}(\boldsymbol{\Sigma})$ equals $\frac{\lambda_j}{\mathrm{tr}(\boldsymbol{\Sigma})}$. According to the properties of matrix trace, $\mathrm{tr}(\boldsymbol{\Sigma}) = \sum_i \lambda_i$, so the variance contribution rate of factor F_j to $\mathrm{tr}(\boldsymbol{\Sigma})$ equals $\frac{\lambda_j}{\mathrm{tr}(\boldsymbol{\Sigma})} = \frac{\lambda_j}{\sum_j \lambda_j}$. After a standardization of variables by $\tilde{\boldsymbol{z}} = \left(\frac{z_1 - \bar{z}_1}{\sqrt{\sigma_{11}}}, \cdots, \frac{z_{N+M+B} - \bar{z}_{N+M+B}}{\sqrt{\sigma_{N+M+B,N+M+B}}}\right)'$, where $\bar{z}_i$ and σ_{ii} are the sample mean and standard deviation of variable z_i, respectively, the total variance satisfies

$$\mathrm{tr}\left(\widetilde{\boldsymbol{\Sigma}}\right) = \dim\left(\widetilde{\boldsymbol{\Sigma}}\right) = N + M + P, \tag{A4}$$

so $\frac{\tilde{\lambda}_j}{\mathrm{tr}(\widetilde{\boldsymbol{\Sigma}})} = \frac{\tilde{\lambda}_j}{N+M+B}$.